Comparison of the Elementary-Effects Method and the Coefficient-of-Importance using a One-dimensional Cooling Flow Network

10/11/2019 - Barbara Fiedler

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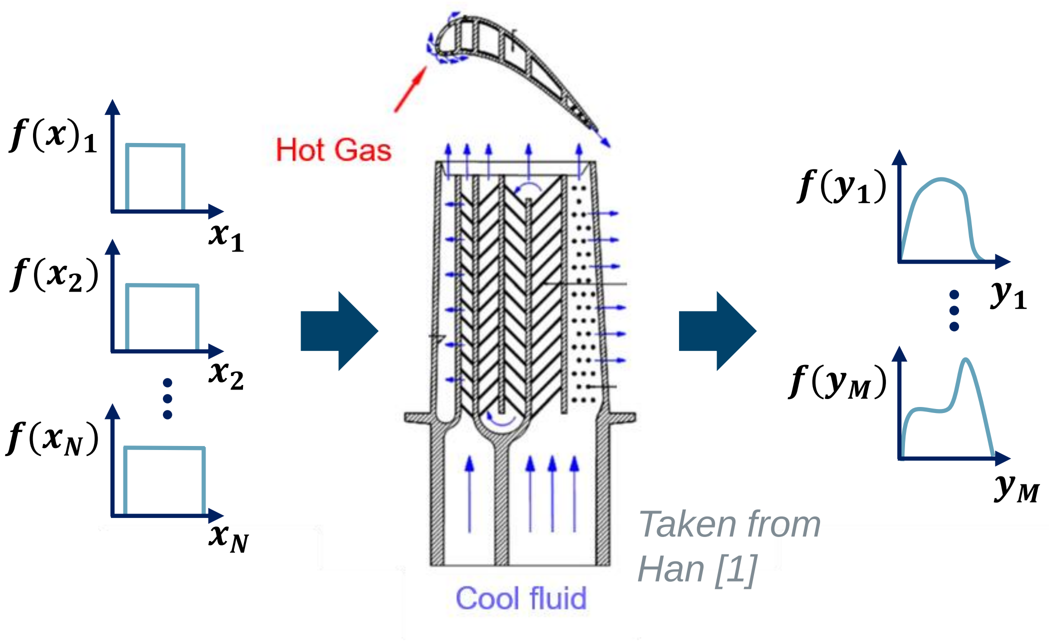
Motivation

- In modern High-Pressure-Turbines the temperature in the main gas path exceeds the tolerable material temperature
- Therefore efficient cooling of turbine blades and vanes is needed with bleed air taken from the compressor
- Better understanding of the sensitivities of the cooling systems to geometric design parameters can help designing more efficient cooling systems
    - ➢ **Comparison of two stochastic methods to quantify the importance of design parameters**



Taken from Han [1]

Importance Ranking of the design parameters

One-Factor-at-Time Method → Elementary-Effects Method

Analysis of Variance → Coefficient-of-Importance

Agenda

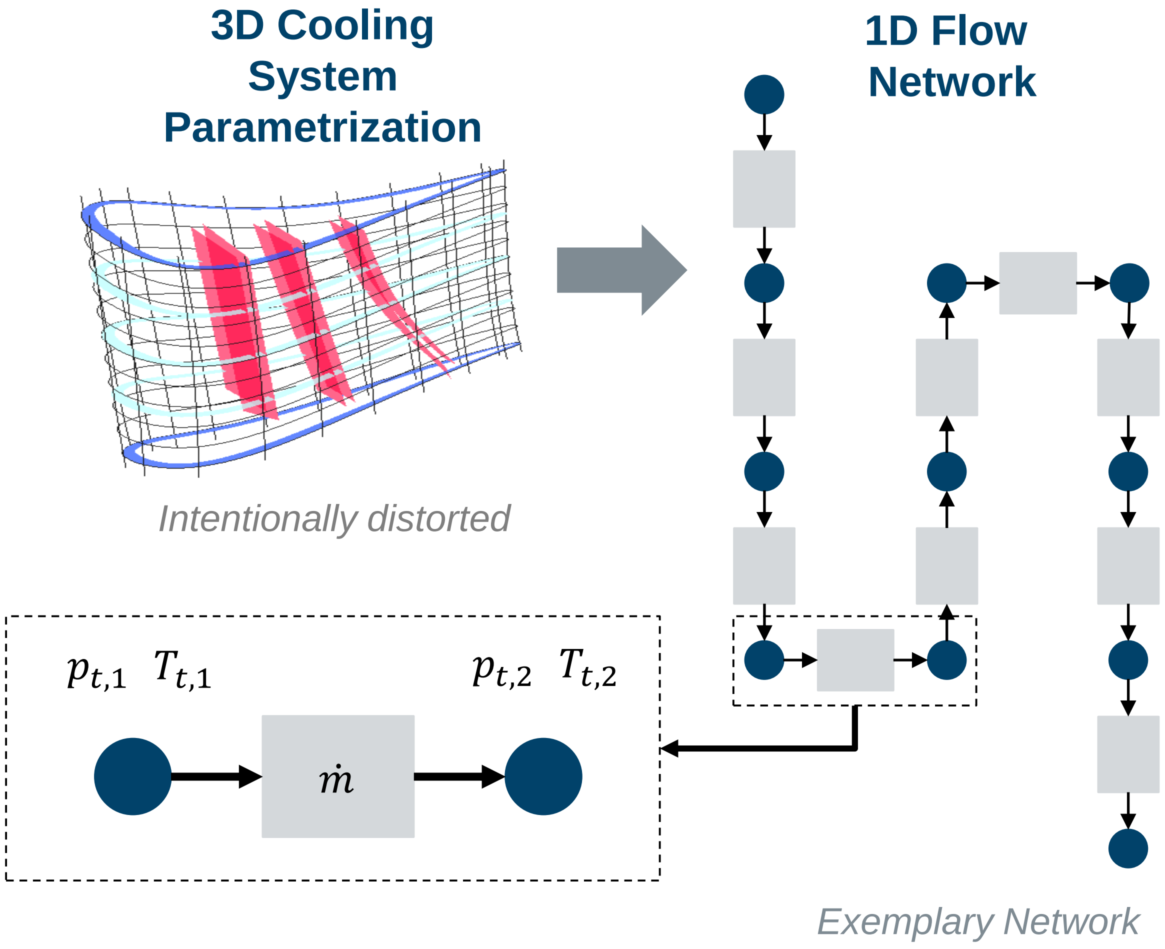
- Deterministic Model
- Parametrization of the Cooling System
- Introduction of probabilistic methods
- Numerical Setup of the Analysis
- Evaluation of the Sensitivity Analysis
- Summary and Outlook

Deterministic Model
Numerical Simulation of the cooling flow

Mesh Generation
- Conversion of the 3D model into simplified 1D flow network
- Segmentation of the 3D system into characteristic flow elements connected by nodes

Physical equation system
- Conservation of mass
- Conservation of energy
- Conservation of momentum

Numerical Solver “Inflow” [2]
- Conservation of mass and energy in network nodes
- Differential equations of pipe flow based on Truckenbrodt [3]
- Empirical correlations for pressure loss and heat transfer coefficients



Parametrization of the Cooling System
Deterministic Parametrization

Overview of components in modern cooling systems

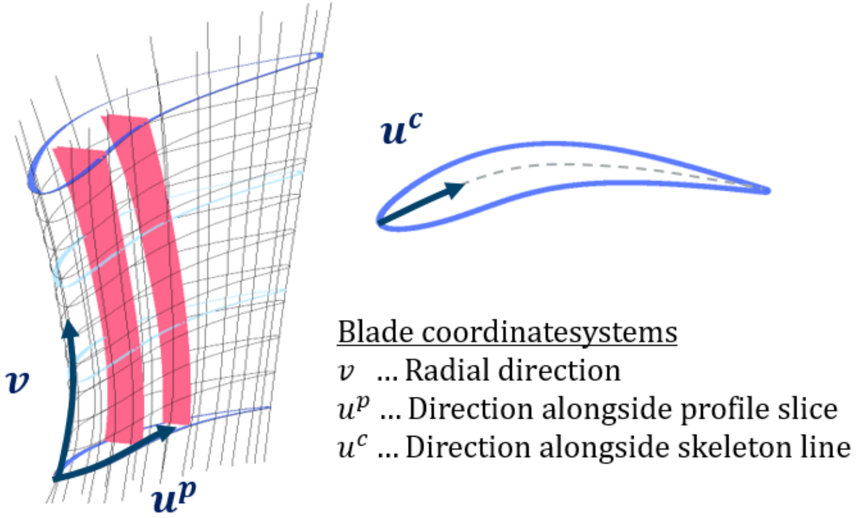
Wall	Webs	Channel Ends	Turbulators	Trailing Edge Slot	Film Cooling

Deterministic geometry definition based on cubic splines used for

- Positioning of components
- Definition of parameter distributions alongside components
  - ➢ Coordinates of distribution points represent design variables

✓ Cubic splines enable definition of curves of any shape

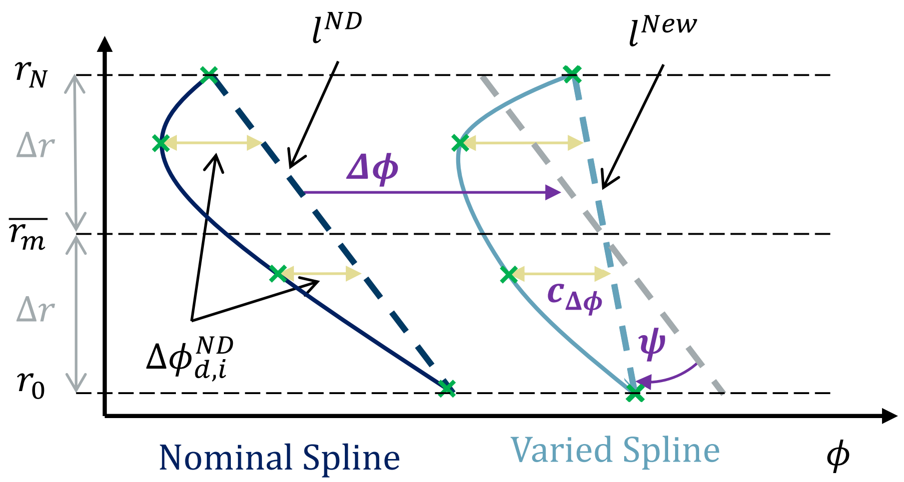
- ❖ Extreme spline deformation due to displacement of points possible
- ❖ Point coordinates not suitable as probabilistic variables



Intentionally distorted

Parametrization of the Cooling System
Development of the Analytical-Coupling-Parametric

Basic principal of the Analytical-Coupling-Parametric for an exemplary spline $\boldsymbol{\phi(r)}$



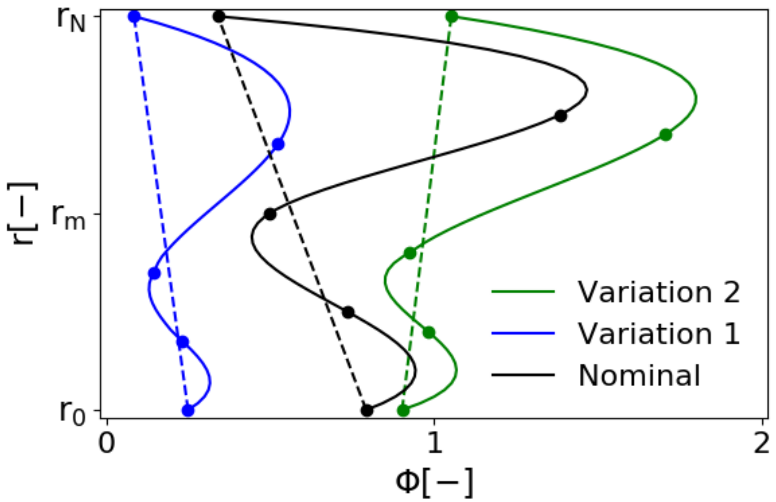
Basic principal

- Introduction of independent, “spline-neutral” parameters constructed from the distribution points of the underlying spline
- A variation of these parameters imposes an incremental delta to the coordinates of the underlying distribution points
- Therefore the distribution points are not moved independently, but are coupled by analytical functions
  - ➢ Characteristic of the original shape is preserved

Advantages

- Universally applicable regardless of the underlying spline shape
- Control over (selected) constructive criteria
- Easier interpretation of the relationships between design variables and resulting system response

Parametrization of the Cooling System
Development of the Analytical-Coupling-Parametric



Symbol	Function
ψ	Rotation of the spline
ϕ_m	Reference position of spline
Δr_{max}	Movement in radial direction (applied as linear functions with $\Delta r_m = \Delta r_{max}$ and $\Delta r_0 = \Delta r_N = 0$)
$c_{\Delta\phi}$	Spline-Line-Distance-Scaling-Factor
$\Delta\phi_{max}$	Maximum Spline-Line-Distance
ϕ_{max}	Maximum spline value
ϕ_{min}	Minimum spline value

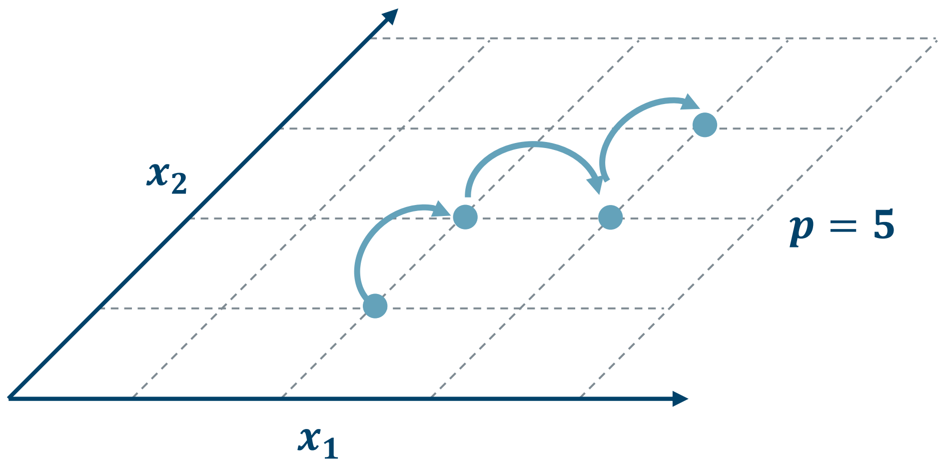
Control of limits ϕ_{max}, ϕ_{min}, used to construct linear function $\Delta\phi(\phi)$, as alternative to parameters ϕ_m, $c_{\Delta\phi}$ and $\Delta\phi_{max}$

Nomenclature used in the following

$\psi^{w1,th}$ … Rotation angle of thickness distribution of web 1

$\left(\frac{w}{h}\right)_{max}^{c1,rib}$ … Maximum rib-width to channel height ratio in channel 1

Introduction of Probabilistic Methods
Elementary-Effects Method



$$\mu_i^* = \frac{1}{N_{sim}} \sum_{j=1}^{N_{sim}} \left| d_i^j \right|$$

Overall effect of a variable

$$\sigma_i = \frac{1}{N_{sim} - 1} \sum_{j=1}^{N_{sim}} \left(d_i^j - \mu_i \right)^2$$

Non-linear or interactive effect

Introduction of the Method

- One-at-Time approach first introduced by Morris [4]
- Provides two measures to investigate the influence of design variables on the output

Procedure for sensitivity analysis [5]

- Partition of each variable range in p discrete levels
- Generation of $N_{sim} = N(N_{var} + 1)$ samples
- Distinction of two samples in single variable
  - ➢ Difference d_i in output quantifies effect of variable

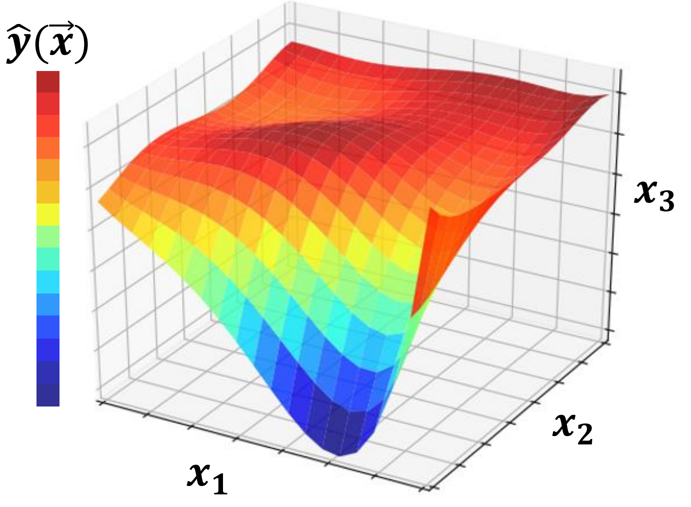
Evaluation of variable influence

- Modified mean effect μ^* [6]
- Standard deviation of effect σ_i

Introduction of Probabilistic Methods

Coefficient-of-Importance

Exemplary Response Surface



$$R^2 = \frac{\sum(\hat{y} - \bar{y})^2}{\sum(y - \bar{y})^2}$$

Variance detected by full regression model

$$CoI_i = R^2 - R^2_{x_i}$$

Variance evoked by variable x_i

Introduction of the Method

- Analysis-of-Variance based approach [5]
- Quantifies variable importance using polynomial regression models [7]

Procedure for sensitivity analysis [7, 8]

- Generation of at least $N_{sim} = (N_{var} + 1)$ samples
- Approximation of the system response $\hat{y}$
- Quantification of full regression model quality R^2
  - ➢ Reduced regression model neglecting a variable quantifies the variable importance

Evaluation of variable influence

- Coefficient of Importance CoI
- ✓ Additional Analysis of Correlations $\rho_{i,j} = (x_i, x_j)$

Numerical Setup of the Analysis

Definition of the Probabilistic Model

Model Description
- High-Pressure-Turbine Blade of second stage
- Multipass-System with rib-roughened walls

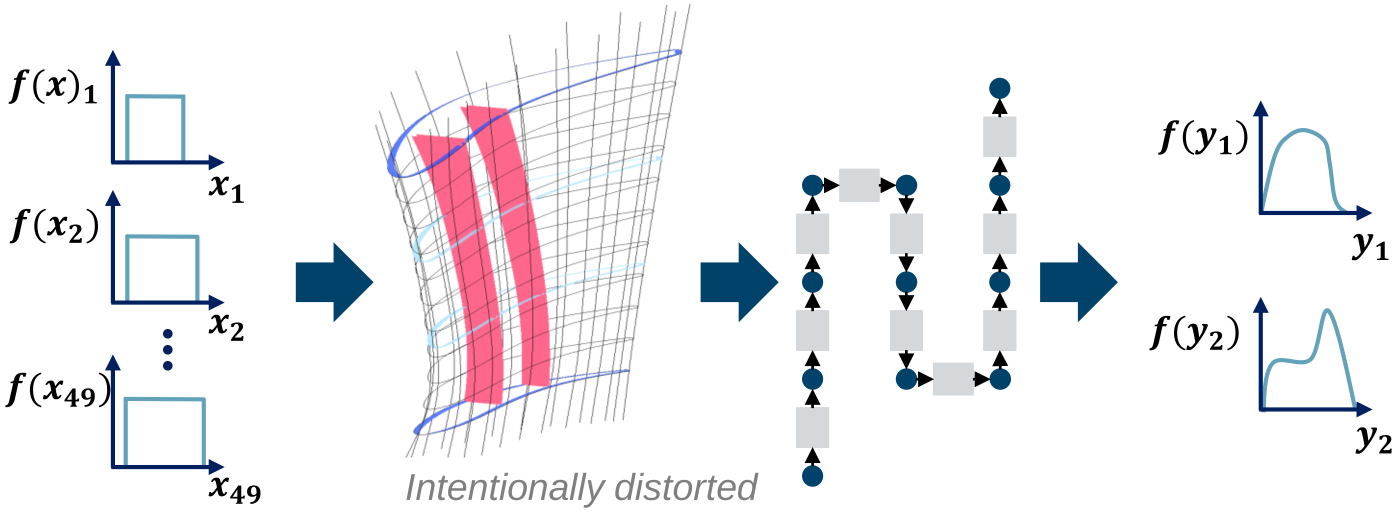
Boundary Conditions of 1D flow solver
- Wall temperature of elements in flow network
- Total pressure and temperature at entry node
- Static pressure at exit node

Variables

Uniformly distributed and uncorrelated

Ranges according to Bunker [9]

Webs, Turbulators, Bends, Pull direction
➢ 49 parameters



Results

Cooling mass flow
$\dot{m}_{cl}$

Pressure rise
$\Delta\tilde{p}_t = \frac{p_{t,out} - p_{t,in}}{p_{t,in}}$

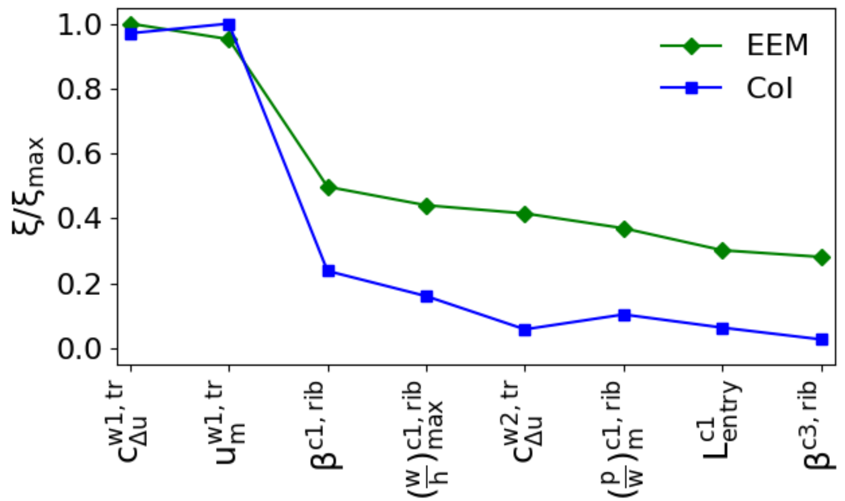
Evaluation of the Sensitivity Analysis

Evaluation of the Sensitivity Analysis
Reference Simulation

Elementary Effects Study	
N_{sim}	500
p^{EEM}	6

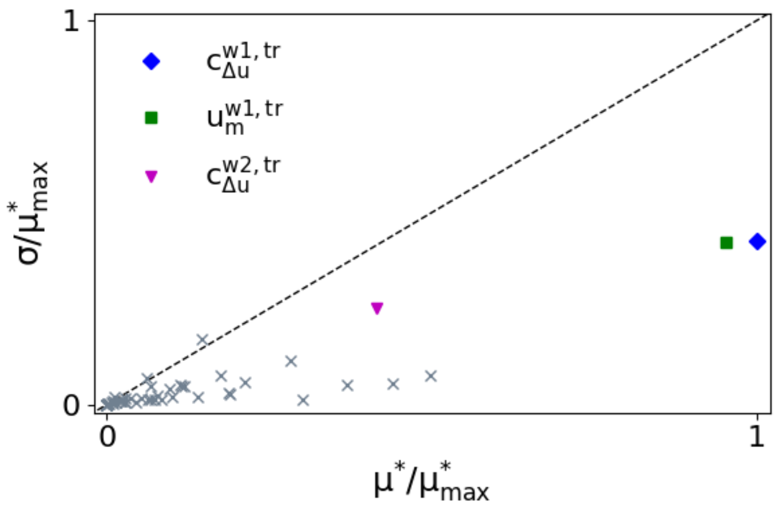
Monte-Carlo-Simulation	
Latin-Hypercube-Sampling	
N_{sim}	500
RSM	1 order ($R^2 = 0.935$)

$\xi = \mathrm{CoI}, \mu^*$



Importance Ranking of the cooling mass flow

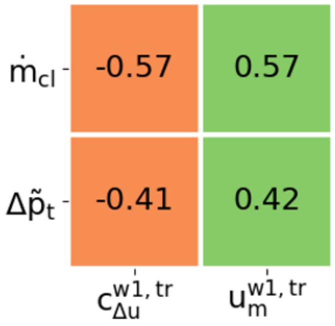
- Good agreement between the ranking of EEM and CoI
- Slight deviations in ranking within small order
- Amount of cooling mass flow dominated by Distance-Scaling Factor $c_{\Delta u}^{w1}$ and reference position u_m^{w1} of the first Web
- No significant interactive effects detectable with EEM



✓ EEM und CoI equivalent for the investigated problem

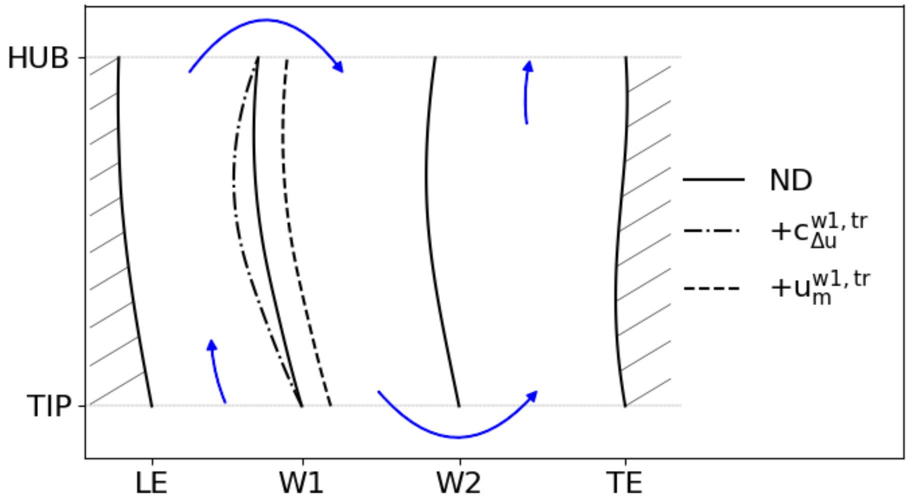
Evaluation of the Sensitivity Analysis

Reference Simulation



Extraction of the Correlation-Coefficient-Matrix (Spearman)

Schematic view of flow channels



Elementary Effects Study	
N_{sim}	500
p^{EEM}	6

Monte-Carlo-Simulation	
Latin-Hypercube-Sampling	
N_{sim}	500
RSM	1 order ($R^2 = 0.963$)

Correlation Analysis

- Flow cross section in nominal design slightly smaller in first channel
- Impact of $c_{\Delta u}^{w1,tr}$ enlargement on first channel
  - ➢ Reduction of overall cross section
  - ➢ Reinforcement of the convergent-divergent area change
- Impact of $u_m^{w1,tr}$ enlargement
  - ➢ Reduction of the area change in the entire system
- ✓ Adjustment of flow cross sections throughout the system reduces pressure losses and thus increases mass flow

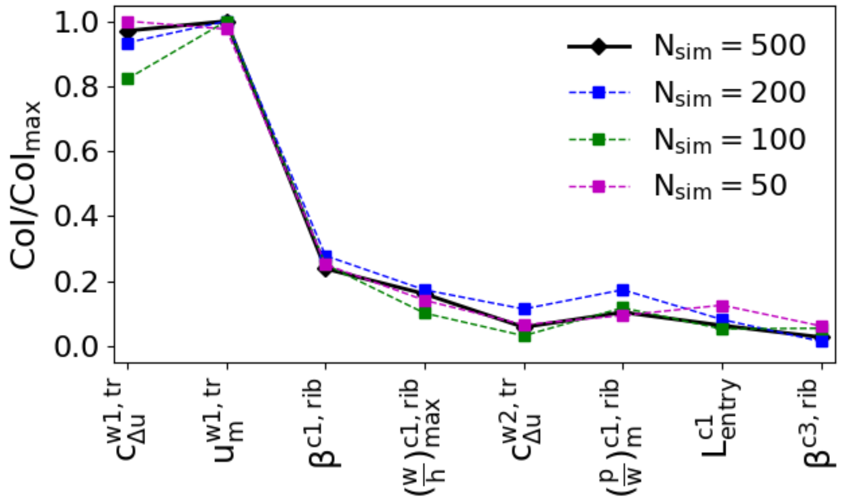
Analysis-of-Variance allows interpretation of physical relations

Evaluation of the Sensitivity Analysis

Influence of the Sampling Size on the Ranking for Cooling Mass Flow

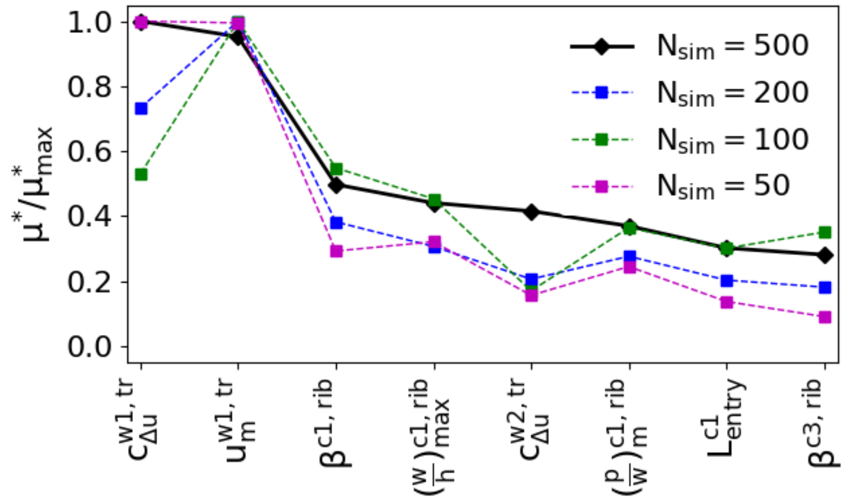
Coefficient-of-Importance

- Good agreement in overall ranking with decreasing sampling size
- Importance of $c_{\Delta u}^{w1,tr}$ slightly depends on sampling size
  - ➢ Well prediction related to linear system behavior



Elementary-Effects-Method

- Importance of $c_{\Delta u}^{w1,tr}$ strongly depends on sampling size
- Clear scattering in ranking after 6th Variable compared to reference simulation
  - ➢ EEM seems more sensitive to population size



- In general good agreement of CoI and EEM for the **same** population size
- Importance ranking with CoI more stable even for very small sampling sizes

Summary

Parametrization of the cooling system

- Deterministic parametric based on cubic splines proved not suitable for probabilistic approach
- Introduction of the Analytical-Coupling parametric for the variation of splines
- Easier interpretation of the physical relationships between geometric variations and resulting system behavior

Sensitivity Analysis of the cooling flow to geometric variations

- Variation of 49 uncorrelated, uniformly distributed variables within ranges taken from literature research
- Cooling mass flow dominated by parameters of first web
- Equalizing the flow cross sections throughout the system reduces pressure losses and increases mass flow

Comparison of Elementary-Effects Method and Coefficient-of-Importance

- Good agreement in the variable importance ranking for sufficiently large population size
- Importance ranking of Elementary-Effects Method allows more detailed investigation of the reason of importance
- Coefficient-of-Importance predicts variable importance more stable with decreasing sampling size for the considered problem and the assumed variable ranges and allows

Outlook

Recommendation for the sensitivity analysis of cooling systems

- Examination of the sensitivities initially using variance-based approaches
  - Quantification of variable importance based on the Coefficient-of-Importance
  - Analysis of correlations to detect physical relationship between design parameters and the system response
- Additional investigation with the Elementary-Effects Method
  - In case the system behavior does not follow a polynomial regression model
  - Distinction between direct and interactive effect of a design variable is required

Motivation for future studies

- Comparison of EEM and CoI for problems with strong interactions between the variable input parameters
- Comparison of Sobol-Indices with EEM and CoI considering the detected sensitivities and numerical effort
- Sensitivity analysis of cooling systems with higher complexity such as systems including film and trailing edge cooling
- Investigation of differences between the sensitivities of vane and blade cooling systems

Thank you for your attention

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