Introducing efficient uncertainty analysis in industrial virtual product development

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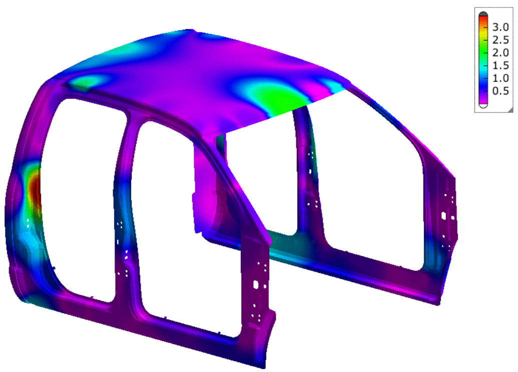

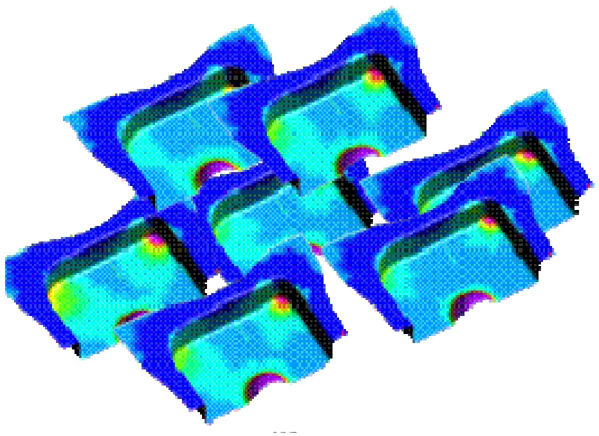

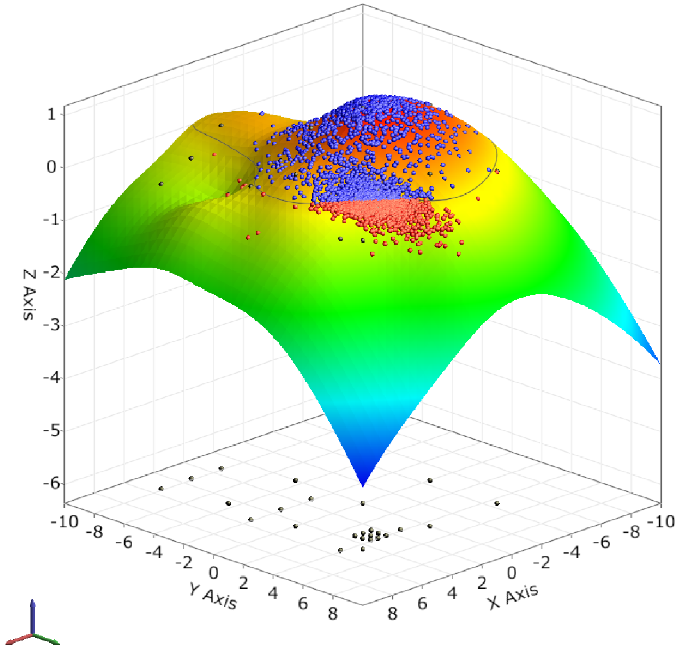

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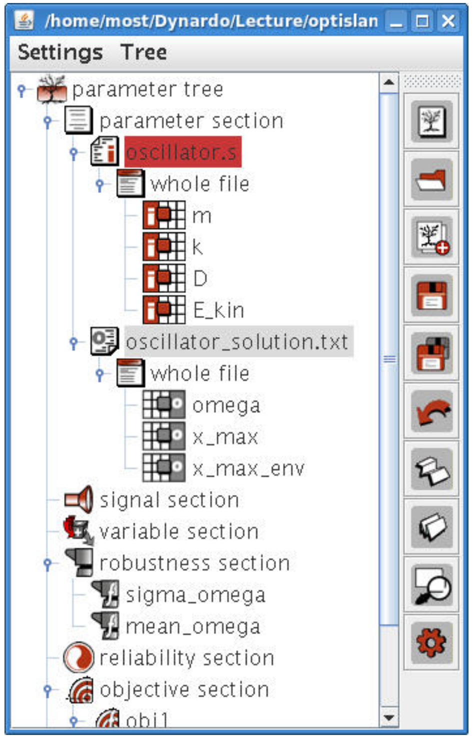
Challenges in Virtual Prototyping

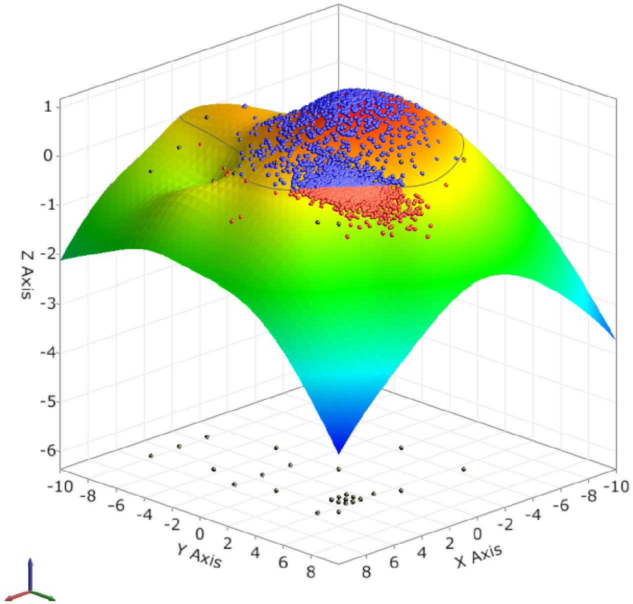
- Virtual prototyping is necessary for cost efficiency
- Test cycles are reduced and placed late in the product development
- CAE-based optimization and CAE-based robustness evaluation becomes more and more important in virtual prototyping

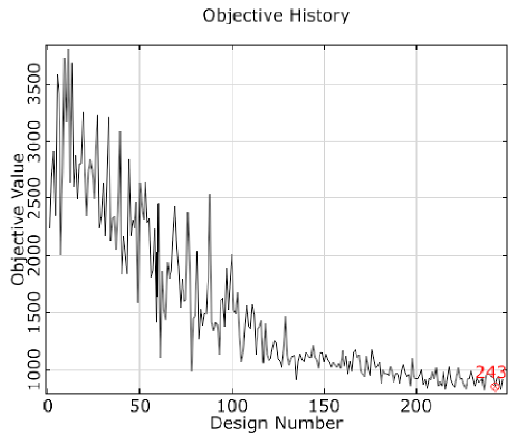
  - Optimization is introduced into virtual prototyping
  - Robustness evaluation is the key methodology for safe, reliable and robust products
  - The combination of optimizations and robustness evaluation will lead to robust design optimization strategies

Excellence of optiSLang

optiSLang is an algorithmic toolbox for sensitivity analysis, optimization, robustness evaluation, reliability analysis and robust design optimization.







optiSLang is the commercial tool that has completed the necessary functionality of stochastic analysis to run real world industrial applications in CAE-based robust design optimizations.

optiSLang development priority: safe of use and ease of use!

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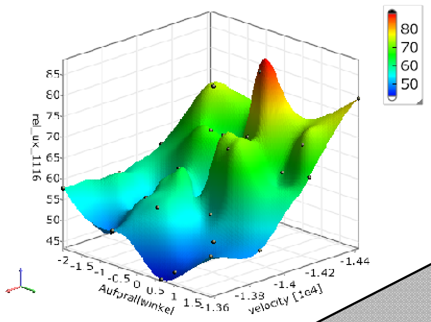
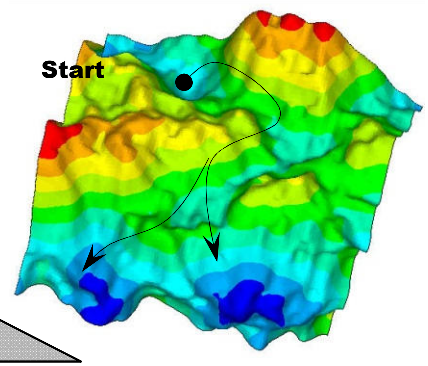
Robust Design Methodology Definition





Robust Design Optimization

Robust Design

Variance based Robustness Evaluation

Probability based Robustness Evaluation, (Reliability analysis)

Optimization

Sensitivity Study

Single & Multi objective (Pareto) optimization

CAE Process (FEM, CFD, MBD, Excel, Matlab, etc.)

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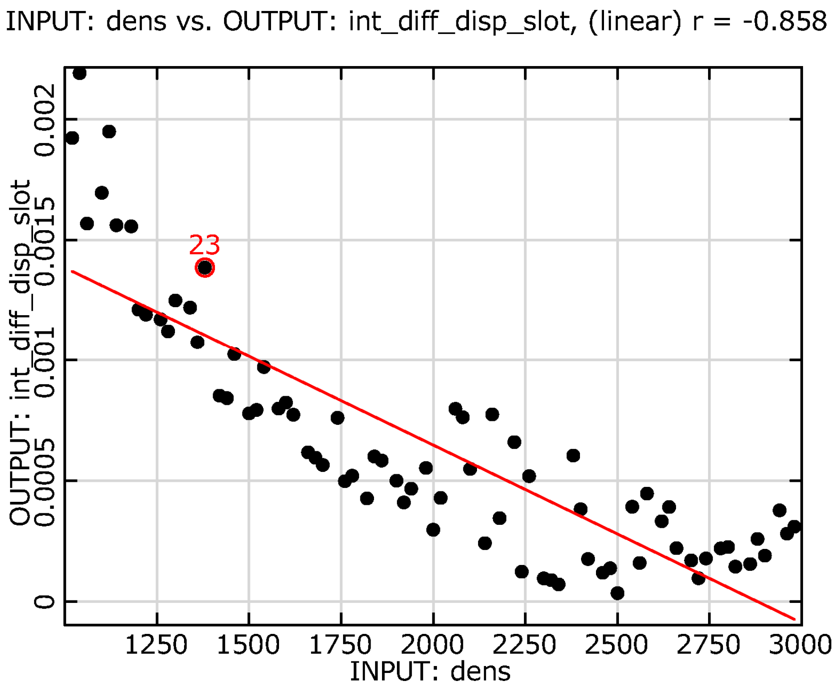
Sensitivity Analysis

optiSLang 3.2

Methods for Sensitivity Analysis

- **Local methods**
  - Local derivatives
  - Standardized derivatives



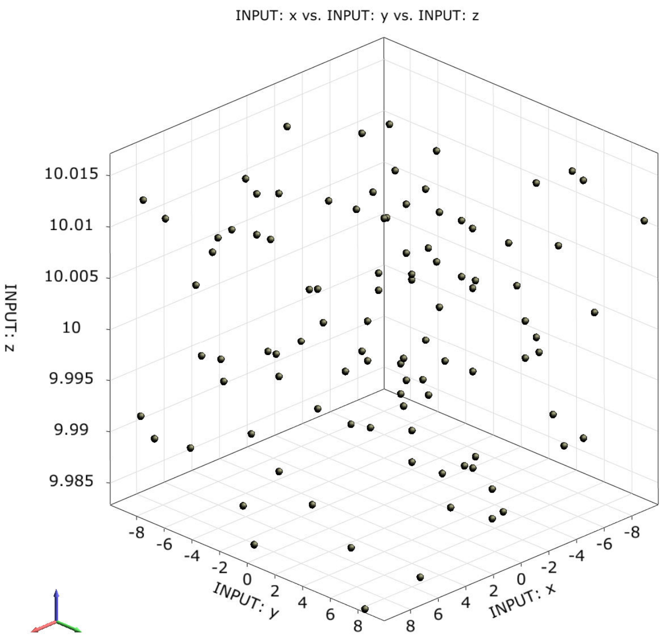
- **Global methods**
  - Scatter plots
  - Coefficients of correlation
  - Rank order correlation
  - Standardized regression coefficients
  - Reduced polynomial models
  - Advanced surrogate models
  - Sobol' indices

Scanning the design/random space

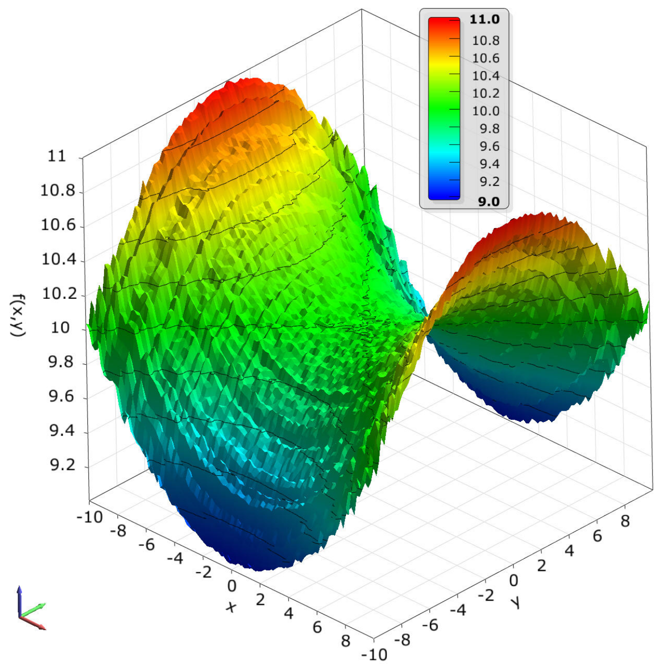
Inputs

$$\left.\begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{matrix}\right\}$$

Sampling



Model evaluation



Outputs

$$\left\}\begin{matrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{matrix}\right.$$

- Joint distribution of inputs is represented by sampling scheme
- Minimum number of samples should represent statistical properties, cover the input space optimally and avoid clustering
- For each design/sample the outputs are calculated/measured

Polynomial regression

- Set of input variables

$$\mathbf{X} = [X_1\ X_2 \dots X_k]^T$$

- Definition of polynomial basis

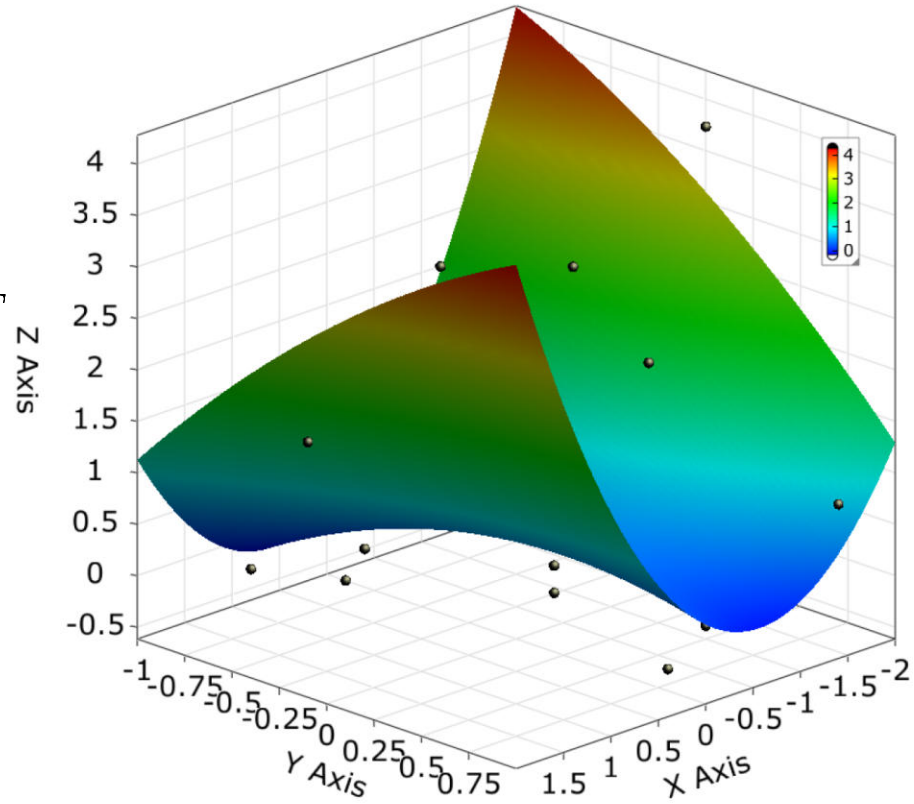
$$\mathbf{p}(\mathbf{x}) = [1\ x_1\ x_2 \dots x_1^2\ x_2^2 \dots x_1 x_2 \dots]^T$$

- Approximation function

$$y(\mathbf{x}) \approx \hat{y}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\hat{\boldsymbol{\beta}}$$

- Least squares solution

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{P}^T\mathbf{P}\right)^{-1}\mathbf{P}^T\mathbf{y}$$



- Number of samples, linear:

$$N \geq 1 + k$$

- Quadratic:

$$N \geq 1 + k + k(k+1)/2$$

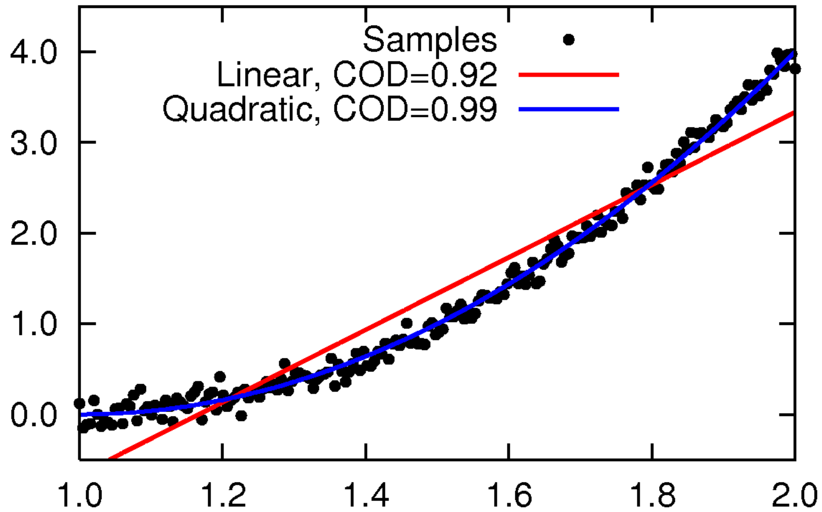
$$\mathbf{P} = \begin{bmatrix} 1 & x_{1,1} & x_{2,1} & \dots & x_{1,1}^2 & x_{2,1}^2 & \dots & x_{1,1}x_{2,1} & \dots \\ 1 & x_{1,2} & x_{2,2} & \dots & x_{1,2}^2 & x_{2,2}^2 & \dots & x_{1,2}x_{2,2} & \dots \\ & & & & \vdots & & & & \\ 1 & x_{1,N} & x_{2,N} & \dots & x_{1,N}^2 & x_{2,N}^2 & \dots & x_{1,N}x_{2,N} & \dots \end{bmatrix}$$

Coefficient of Determination (CoD)

- Fraction of explained variation of an approximated response

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}$$

$$0 \leq R^2 \leq 1$$



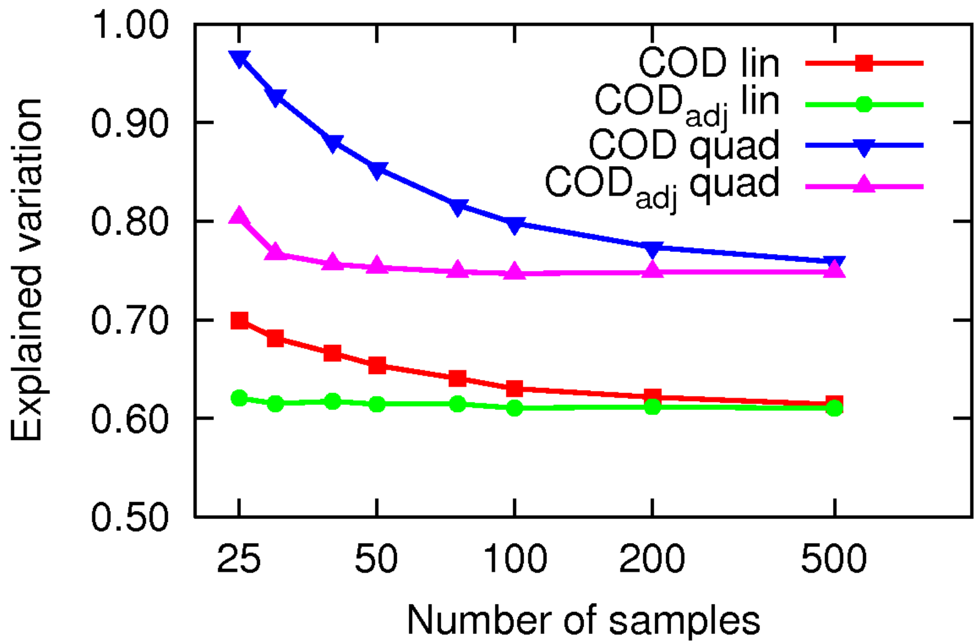
- Explained variation $$SS_R = \sum_{i=1}^{N} (\hat{y}_i - \mu_{\hat{Y}})^2$$

- Total variation $$SS_T = \sum_{i=1}^{N} (y_i - \mu_Y)^2$$

- Unexplained variation $$SS_E = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

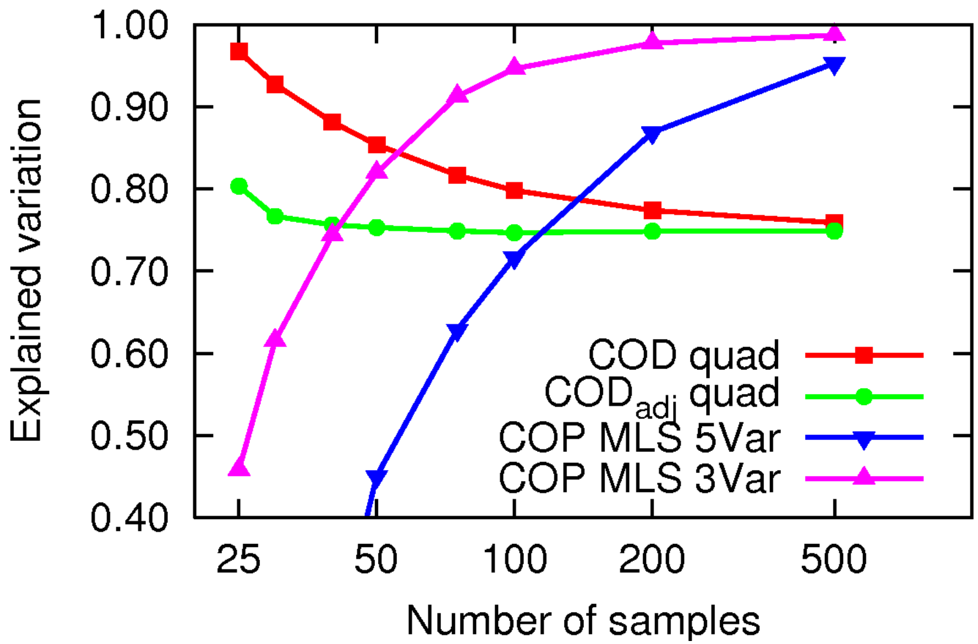
- Adjusted CoD to penalize over-fitting $$R^2_{adj} = 1 - \frac{N-1}{N-p}(1 - R^2)$$

Limitations of CoD



- CoD is only based on how good regression model fits through the sample points, but not on how good is the prediction quality
- Approximation quality is too optimistic for small number of samples
- For interpolation models with perfect fit, CoD is equal to one
- Better approximation models are required for highly nonlinear problems, but CoD works only with polynomials

Coefficient of Prognosis (CoP)



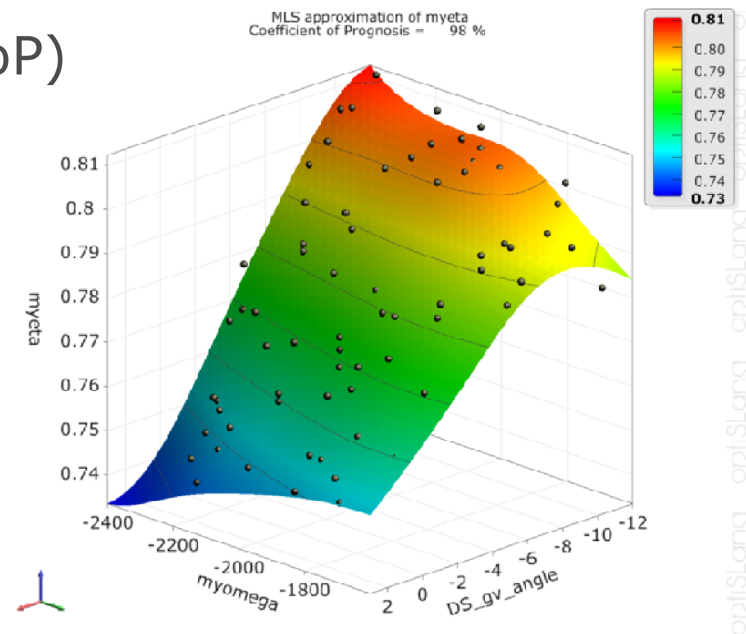
- Fraction of explained variation of prediction
- Estimation of CoP by using a partitioning of available samples

- CoP increases with increasing number of samples
- CoP works for interpolation and regression models
- With Moving Least Squares continuous functions also including coupling terms can be represented
- Prediction quality is better if unimportant variables are removed from the approximation model

Meta-model of Optimal Prognosis (MOP)

- Approximation of solver output by fast surrogate model
- Reduction of input space to get best compromise between available information (samples) and model representation (number of input variables)
- Advanced filter technology to obtain candidates of optimal subspace (significance and CoI filters)
- Determination of most appropriate approximation model (polynomials with linear or quadratic basis, MLS, …, Box-Cox)
- Assessment of approximation quality (CoP)

- **MOP solves three important tasks:**
  - Best variable subspace
  - Best meta-model
  - Determination of prediction quality



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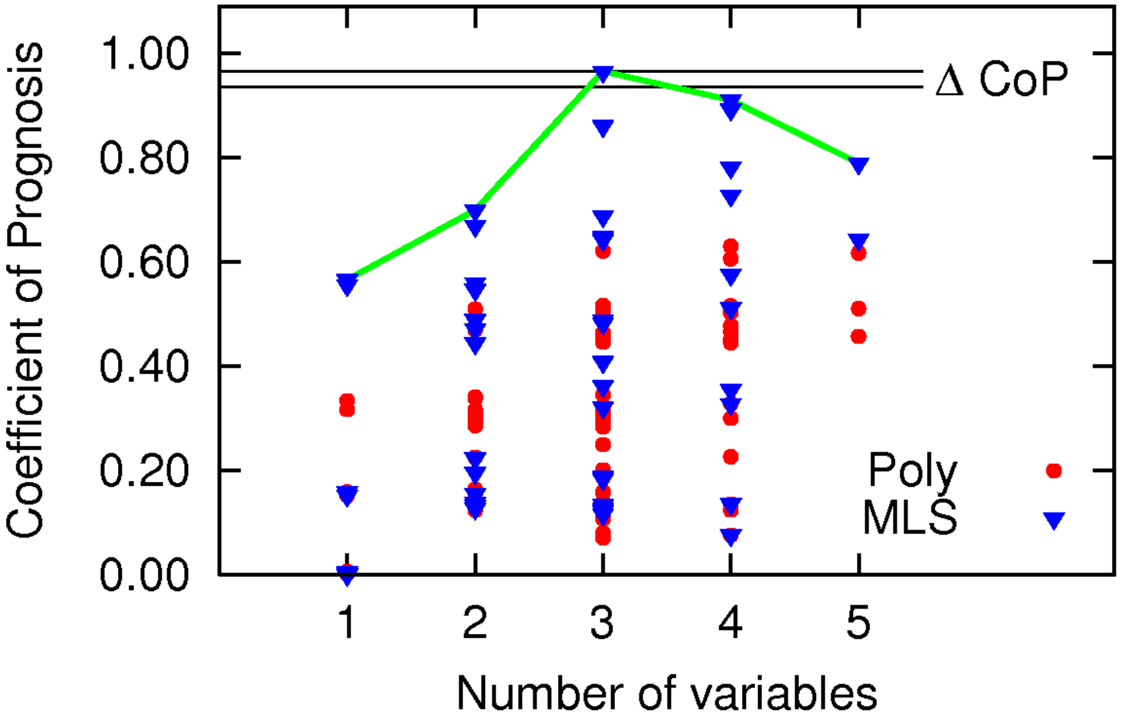
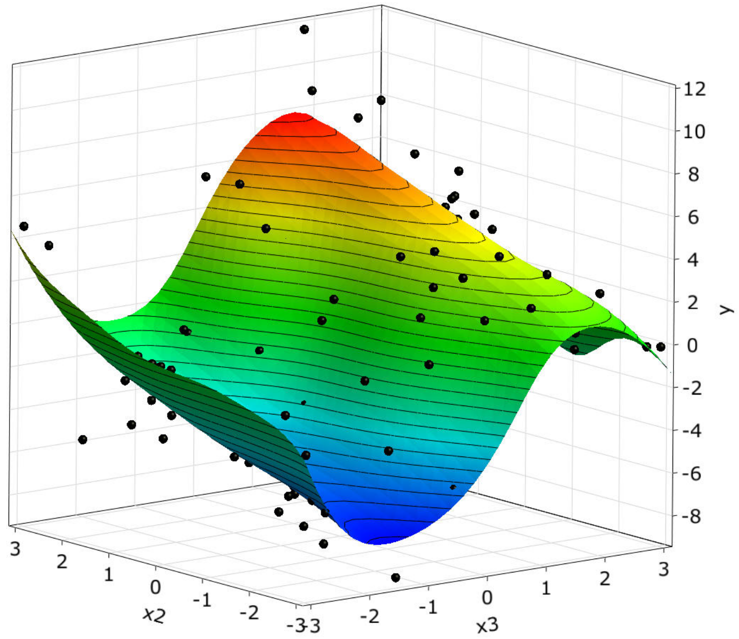
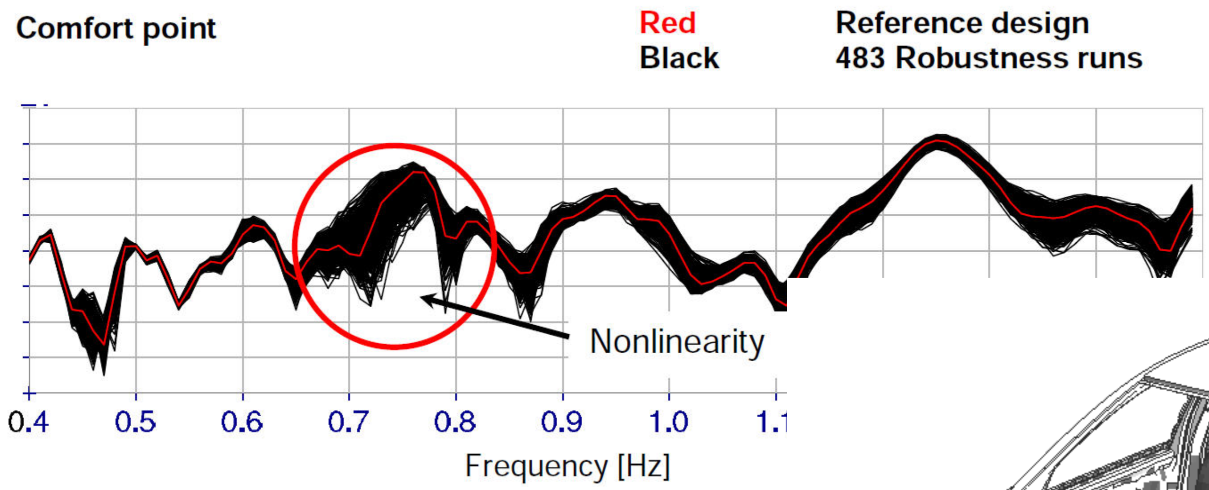
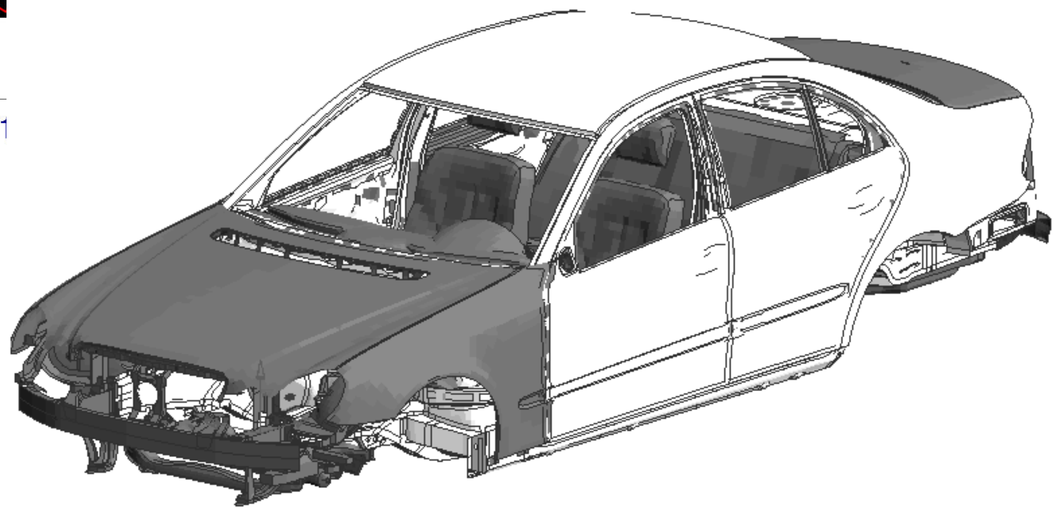




- Advanced filter technology dramatically reduces the number of investigated subspaces to improve efficiency
- Nevertheless, a large number of inputs requires very fast and reliable construction of approximation model

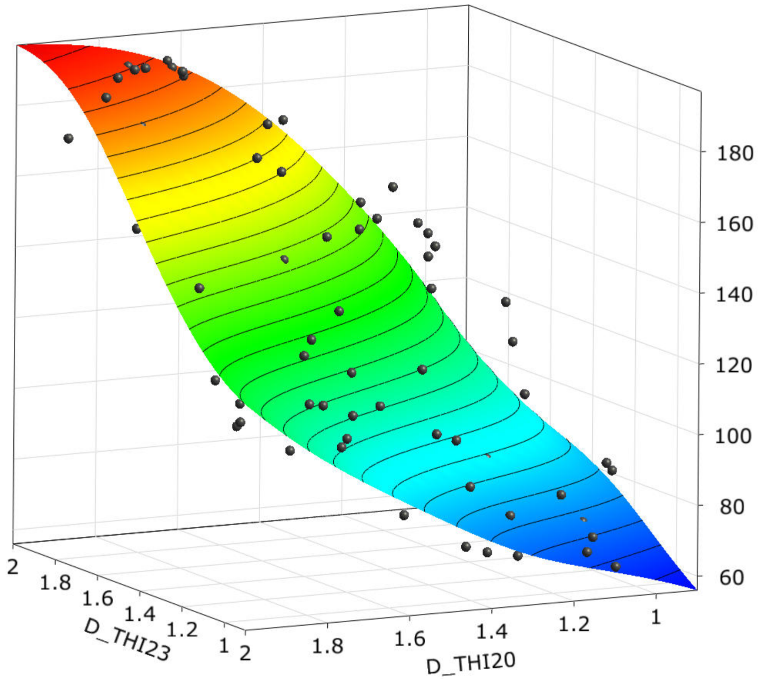
Example: Noise Vibration Harshness

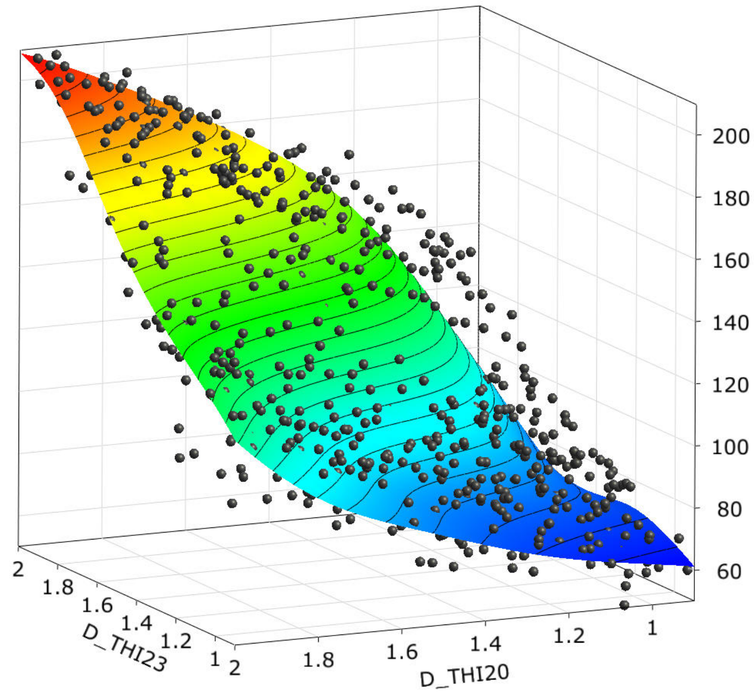




- Input parameters are 46 sheet thicknesses of a car body
- Variation of inputs within a +/- 20% interval
- Output values are sound pressure levels at certain frequencies
- Already single solver run is very time consuming

Example: Noise Vibration Harshness

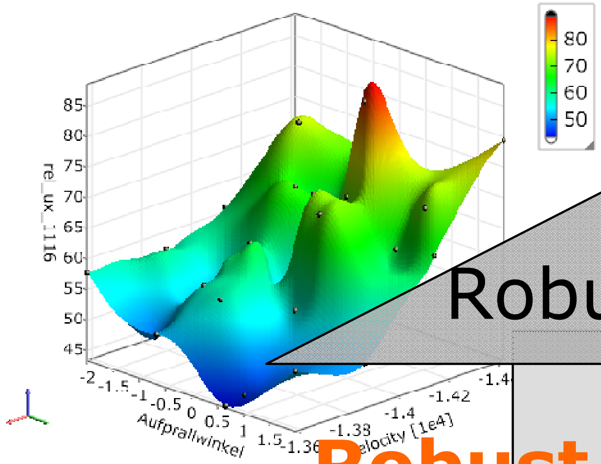


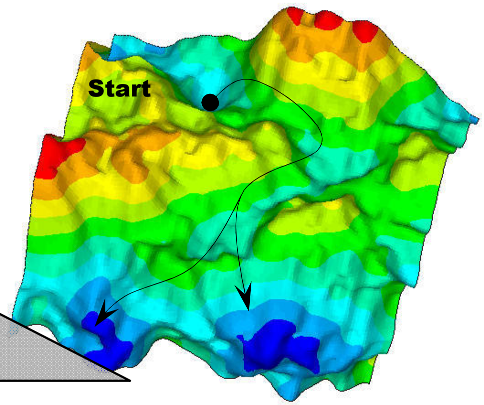


Samples	100	200	400	600	800
Full model	90.9%	91.7%	95.7%	96.3%	96.9%
D_THI5	-	-	2.4%	2.3%	2.7%
D_THI6	6.0%	5.3%	8.2%	8.3%	8.7%
D_THI20	41.3%	42.7%	42.3%	43.4%	42.2%
D_THI23	49.1%	48.0%	50.7%	51.0%	53.8%

Robustness Analysis

Robust Design Methodology Definition





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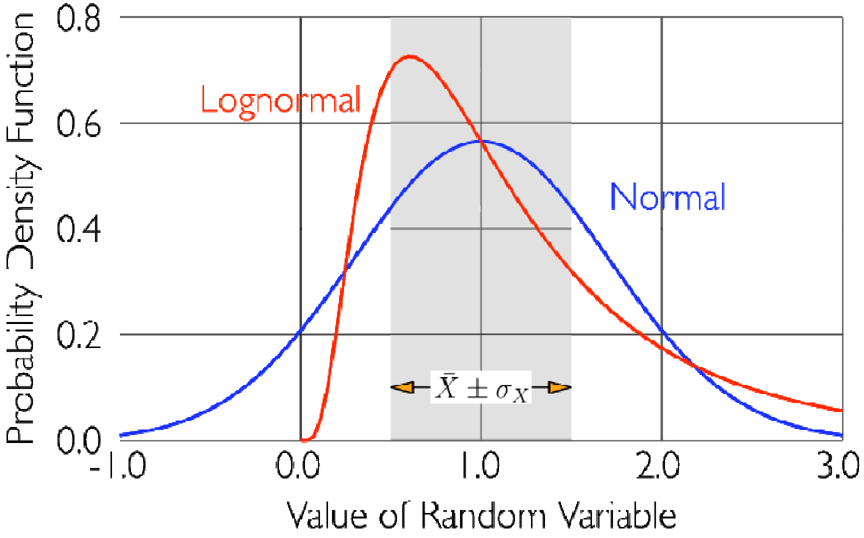
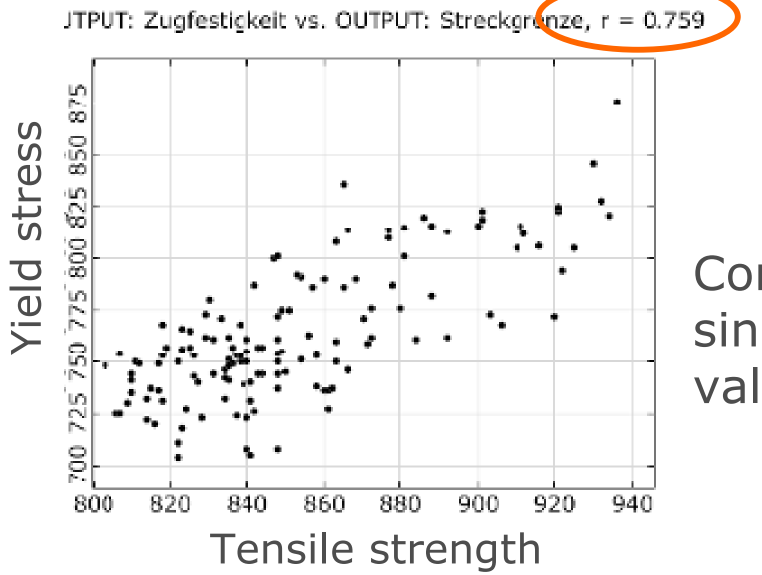
Definition of Uncertainties

- Translate know how about uncertainties into proper scatter definition

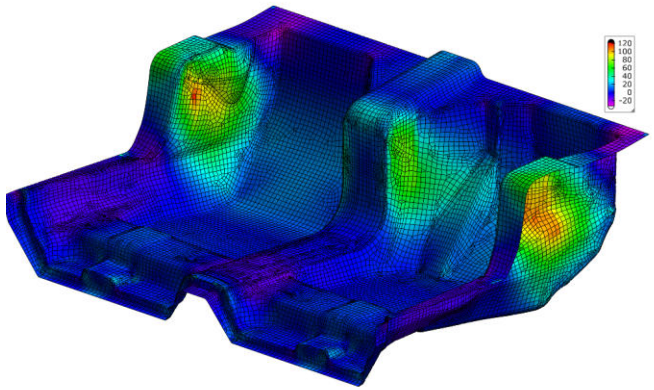


Distribution functions define variable scatter



Correlation of single uncertain values

Correlation is an important characteristic of stochastic variables.



Spatial Correlation = random fields

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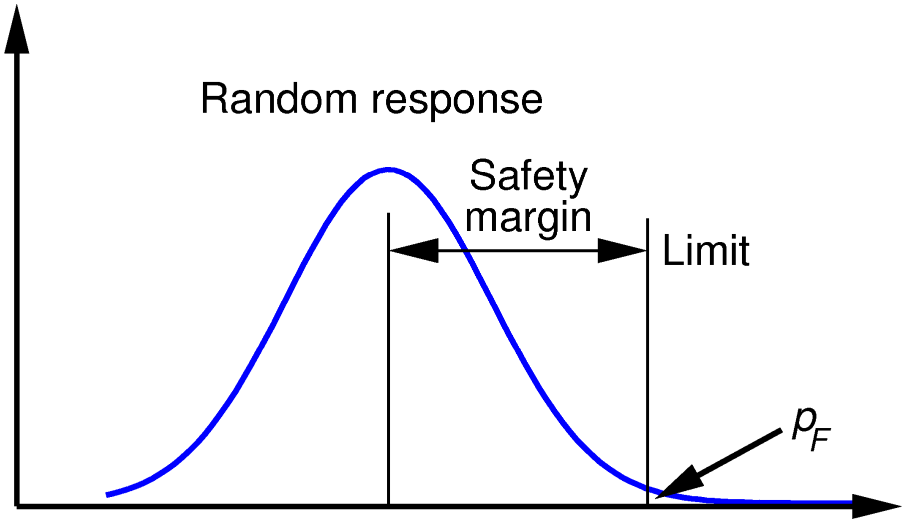
How to Define Robustness of a Design

- Intuitively: The performance of a robust design is largely unaffected by random perturbations
- Variance indicator: The coefficient of variation (CV) of the objective function and/or constraint values is smaller than the CV of the input variables
- Sigma level: The interval mean+/- sigma level does not reach an undesired performance
(e.g. design for six-sigma)
- Probability indicator: The probability of reaching undesired performance is smaller than an acceptable value

Robust Design Optimization

Robustness in terms of constraints



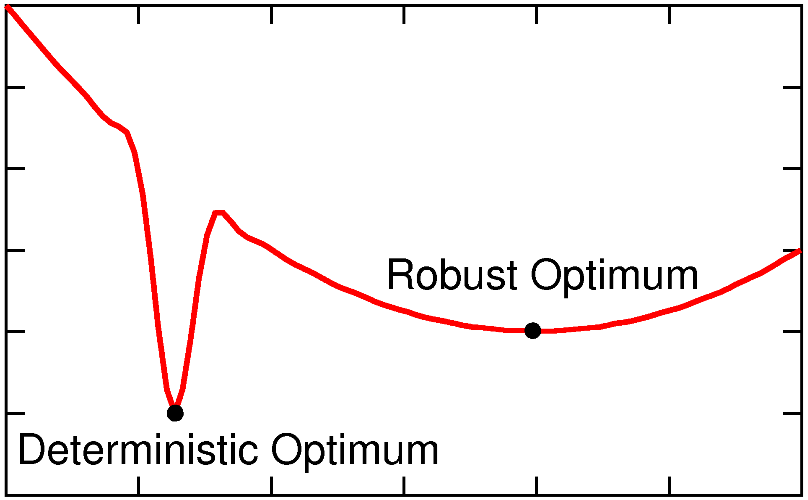
- Safety margin (sigma level) of one or more responses *y*:

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective

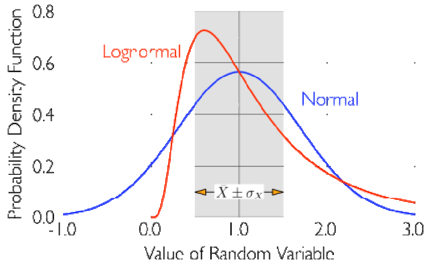


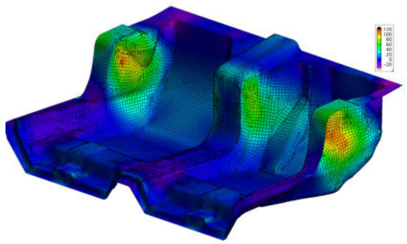
- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

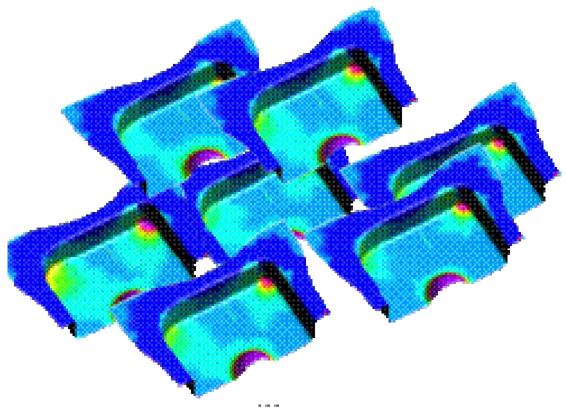
Robustness Evaluation using optiSLang

1) Define the robustness space using scatter range, distribution and correlation

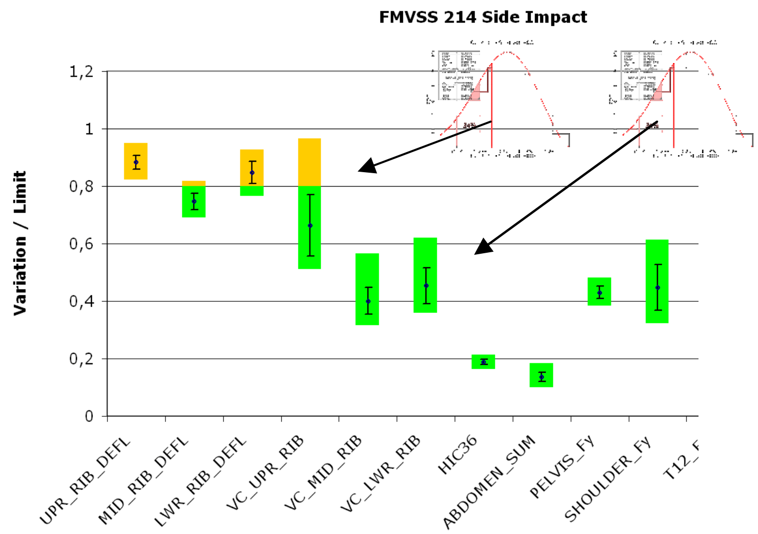




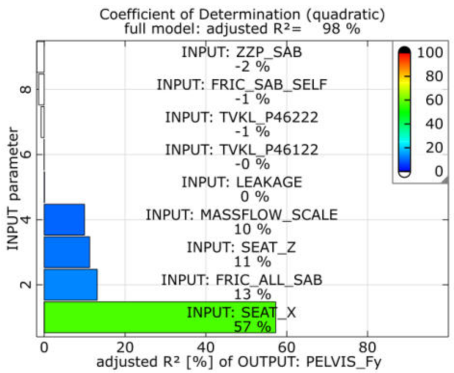
2) Scan the robustness space by producing and evaluating *n* designs



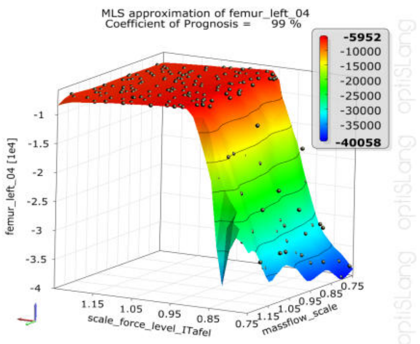
3) Check the variation



4) Check the CoP



5) Identify the most important scattering variables



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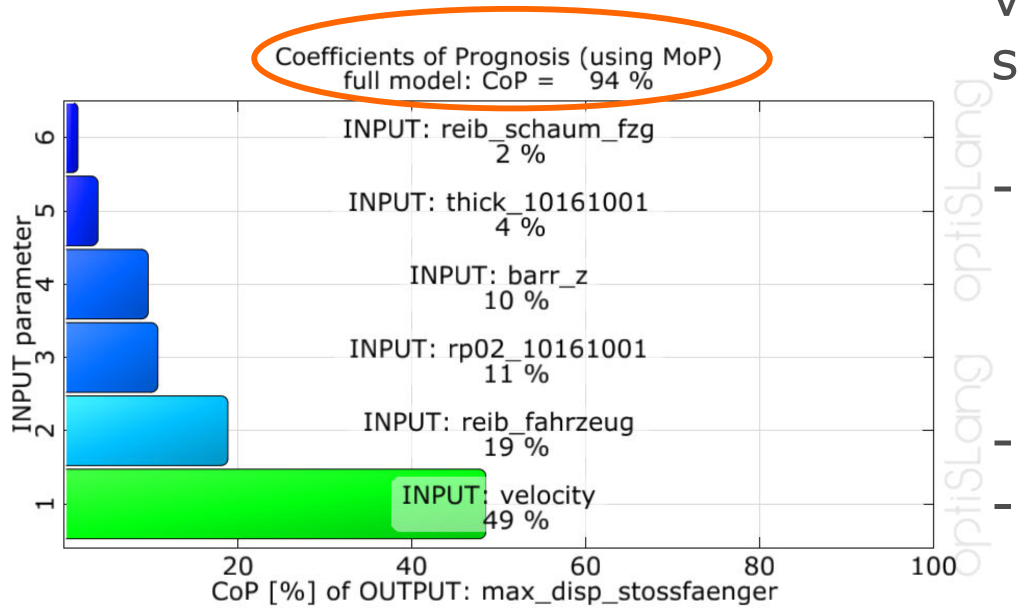
Robustness and Stability of the CAE-Model

Which quantity of „numerical noise“ is acceptable?

⇨ Quantification via coefficients of prognosis (CoP)
⇨ Estimation of numerical noise: 100% - CoP



Experience in passive safety, CFD or crashworthiness tells that result values with lower CoP than 80% show:

- High amount of numerical noise resulting from numerical approximation method (meshing, material, contact,..)
- Problems of result extractions
- Physically instable behavior

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Robustness evaluation as early as possible

Goal: Tolerance check before any hardware exist!

Classical tolerance analysis tend to be very conservative

Robustness evaluation against production tolerances and material scatter (43 scattering parameter) shows:

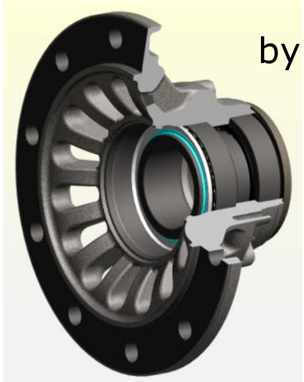
- Press fit scatter is o.k.
- only single tolerances are important (high cost saving potentials)

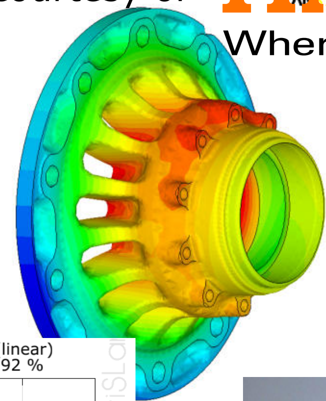
Production shows good agreement!

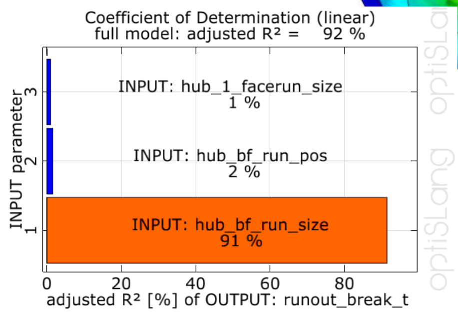
Design Evaluations: 150
solver: ANSYS-optiSLang

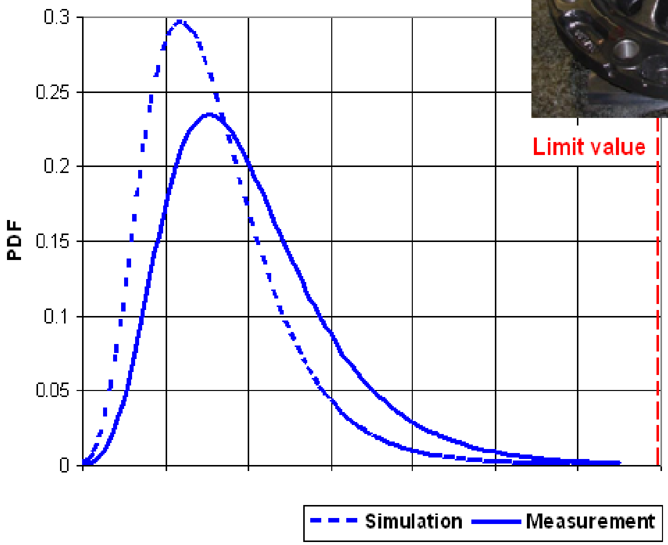
[Suchanek, J.; Will, J.: Stochastik analysis as a method to evaluate the robustness of light truck wheel pack; Proceedings Weimarer Optimierung- und Stochastiktage 6.0, 2009, Weimar, Germany (www.dynardo.de)]

by courtesy of TIMKEN Where You Turn









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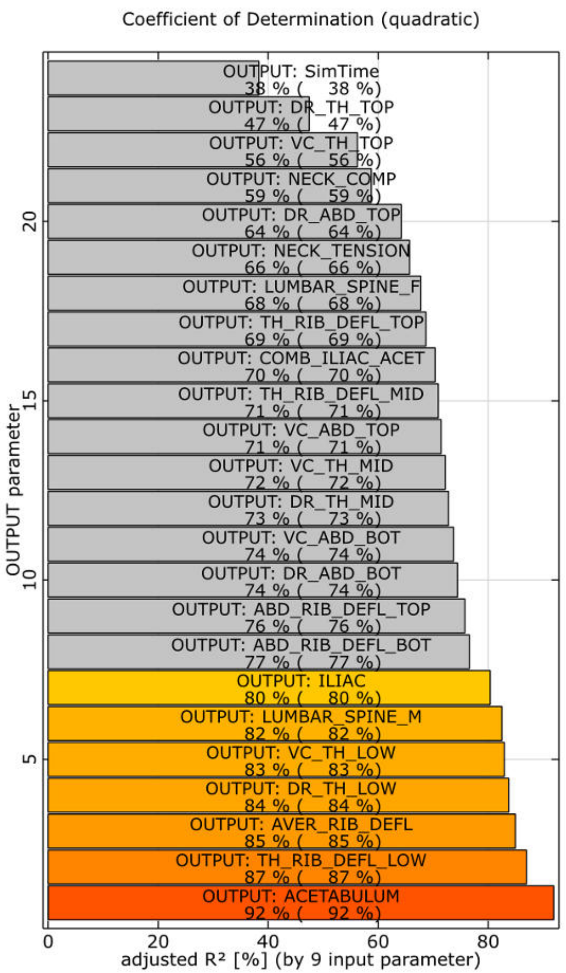
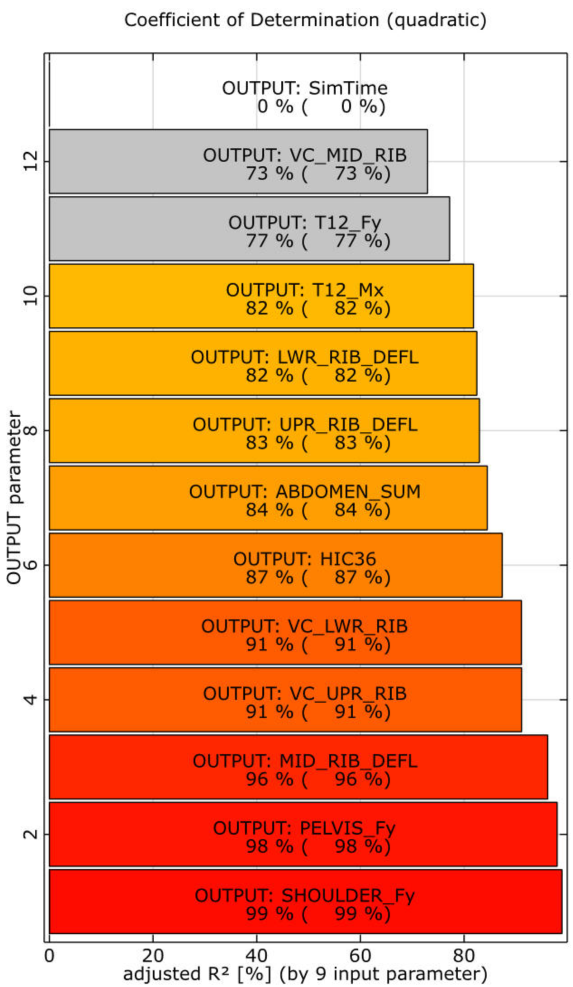
Numerical Robustness Passive Safety

- Comparison of coefficients of determination (CoD) for different FE models (folded airbag/scaled airbag)

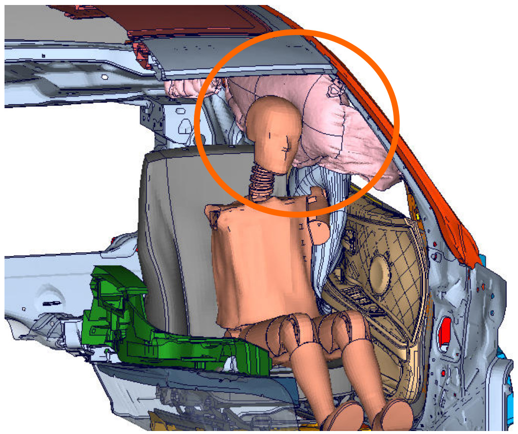


IIHS



FMVSS

The coefficients of determination of the folded airbag analysis show significantly lower values ⇨ in this case it could be shown that the folded airbag does have much more numerical noise than the unfolded!



Robust Design Optimization

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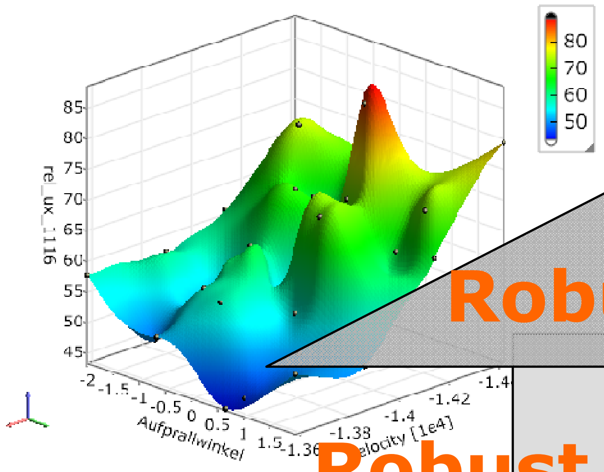

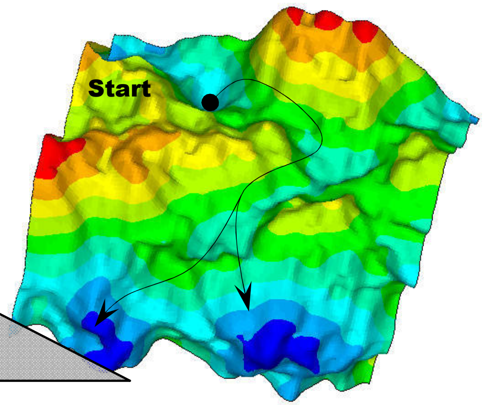

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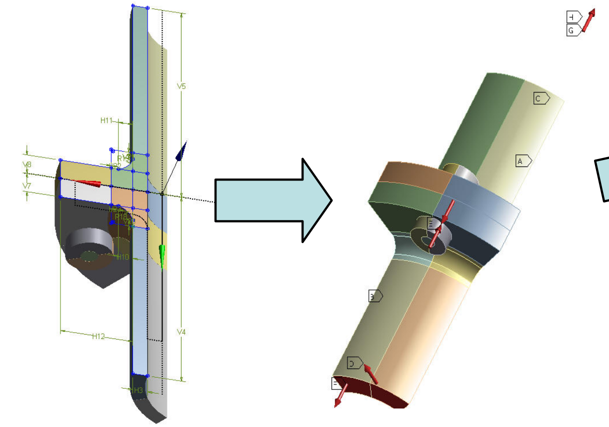
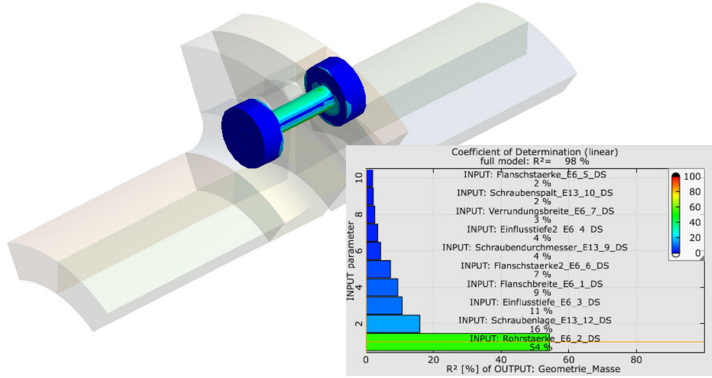
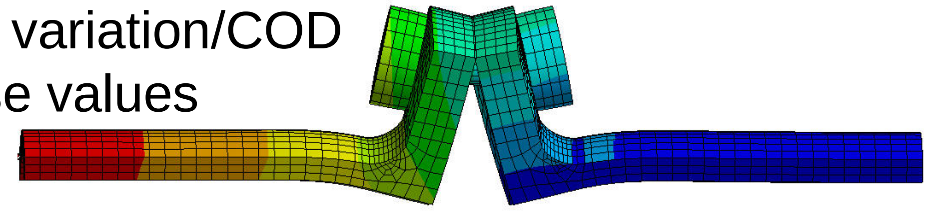
Multidisciplinary Optimization with optiSLang



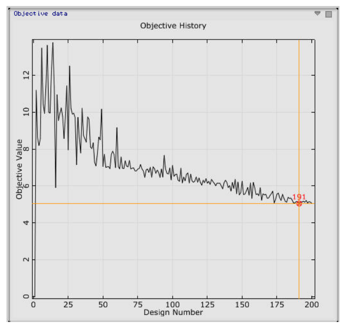
CAD and CAE
Parameter definition



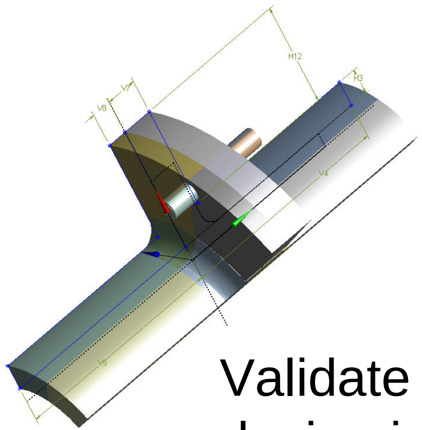
Sensitivity study – identify the most important parameters and check variation/COD of response values

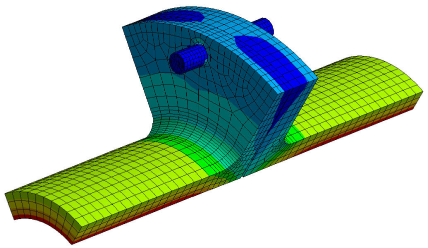


minimize



Define optimization goal and optimize

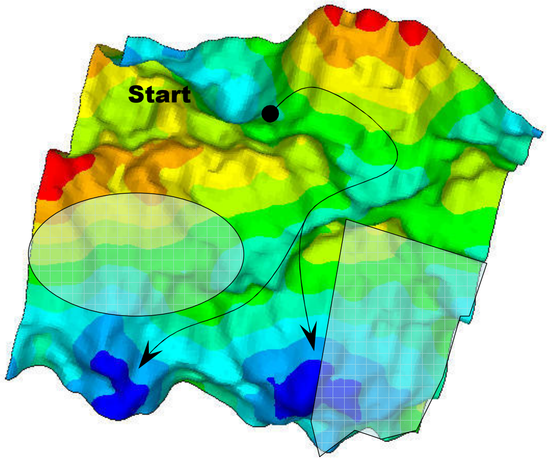




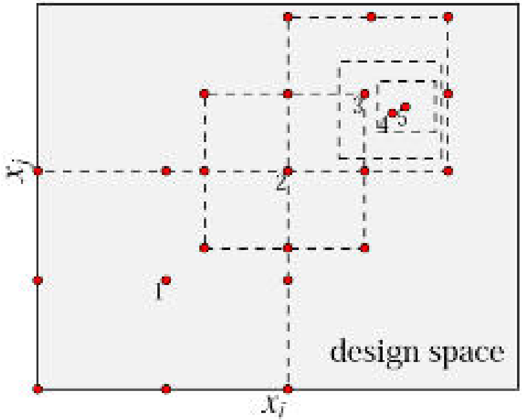
Validate optimized design in CAE and CAD

Optimization Algorithms

Gradient-based algorithms



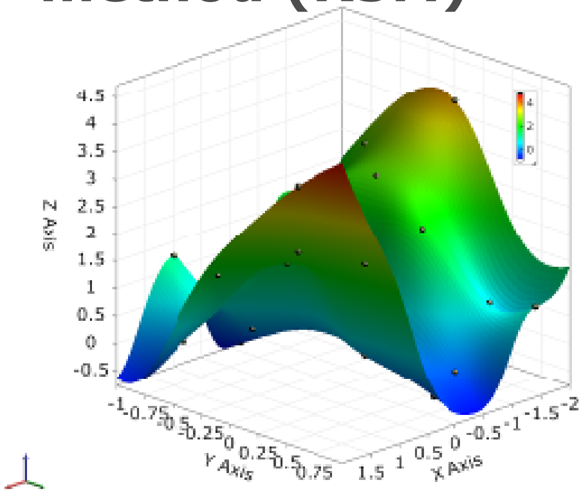
Local adaptive RSM



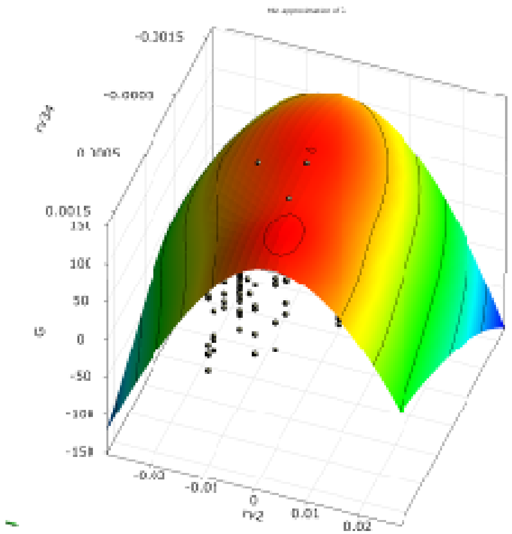
Biologic Algorithms

Genetic algorithms,
Evolutionary strategies &
Particle Swarm Optimization

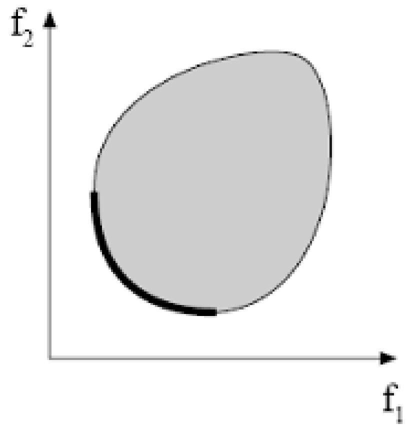
Response surface method (RSM)



Global adaptive RSM

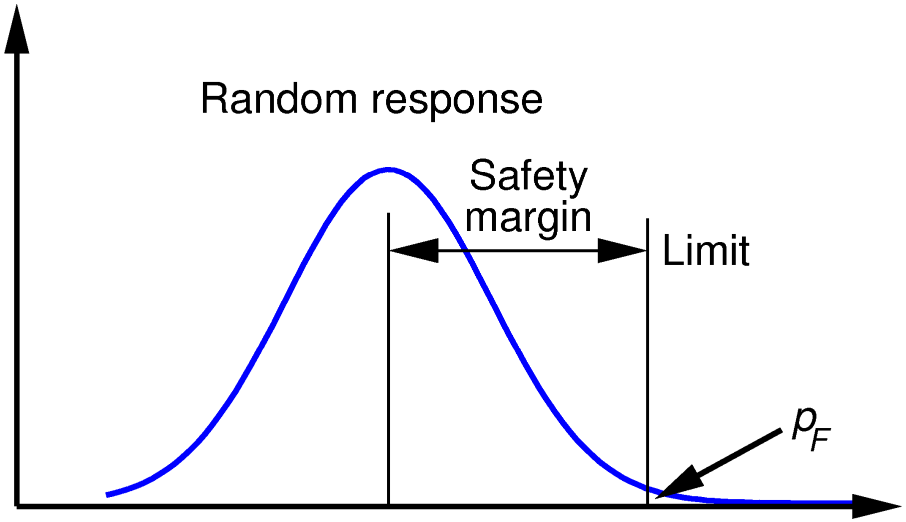


Pareto Optimization



Robust Design Optimization

Robustness in terms of constraints



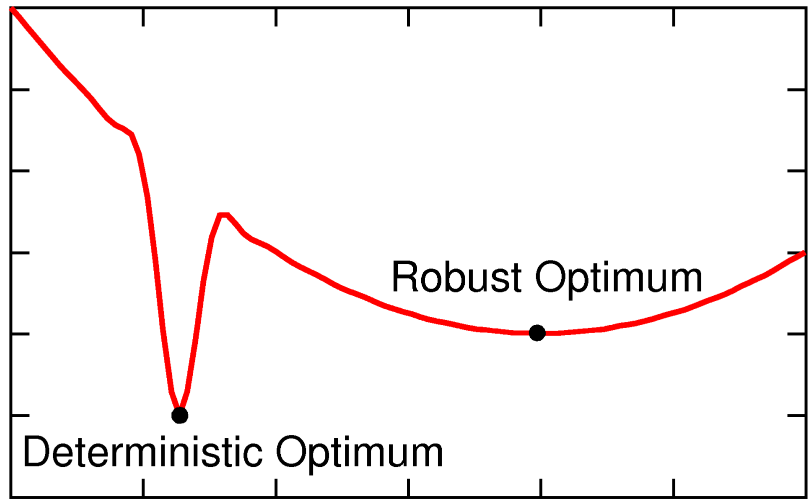
- Safety margin (sigma level) of one or more responses *y*:

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective

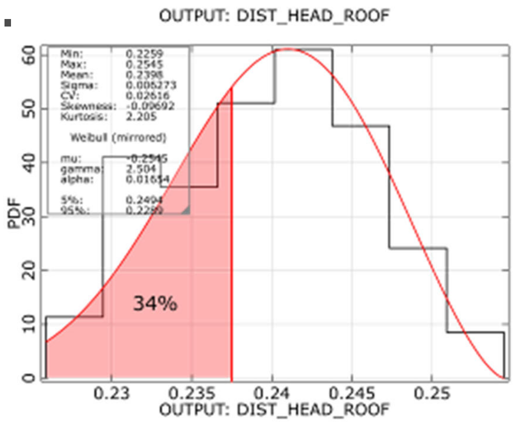


- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

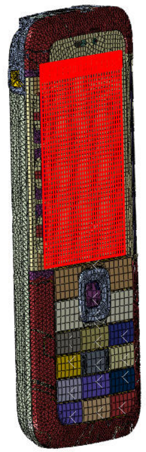
$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

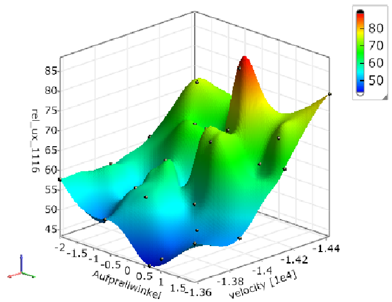
The iterative RDO Procedure

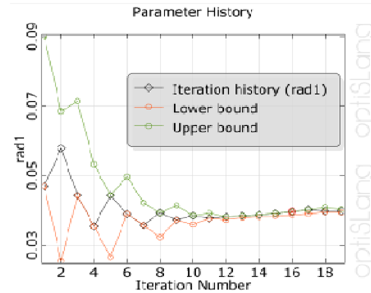
From our experience it is often necessary to investigate both domains, the design space of optimization and the robustness space to be able to formulate a RDO problem. optiSLang offers iterative and automatic RDO flows.

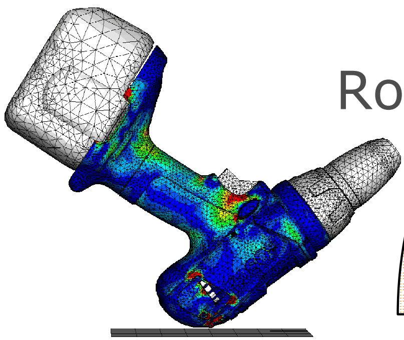


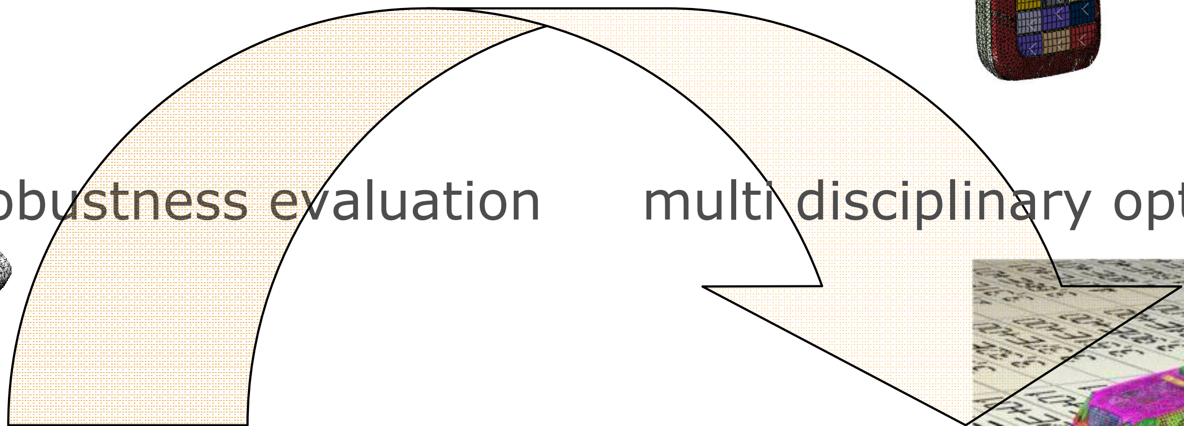
Define safety factors





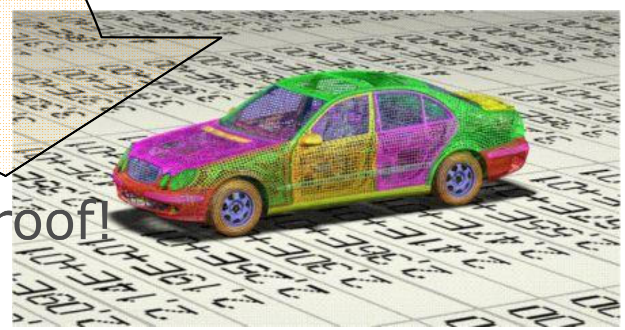






Robustness evaluation

multi disciplinary optimization



Robustness proof!

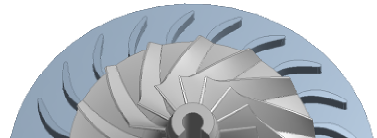
Sensitivity analysis

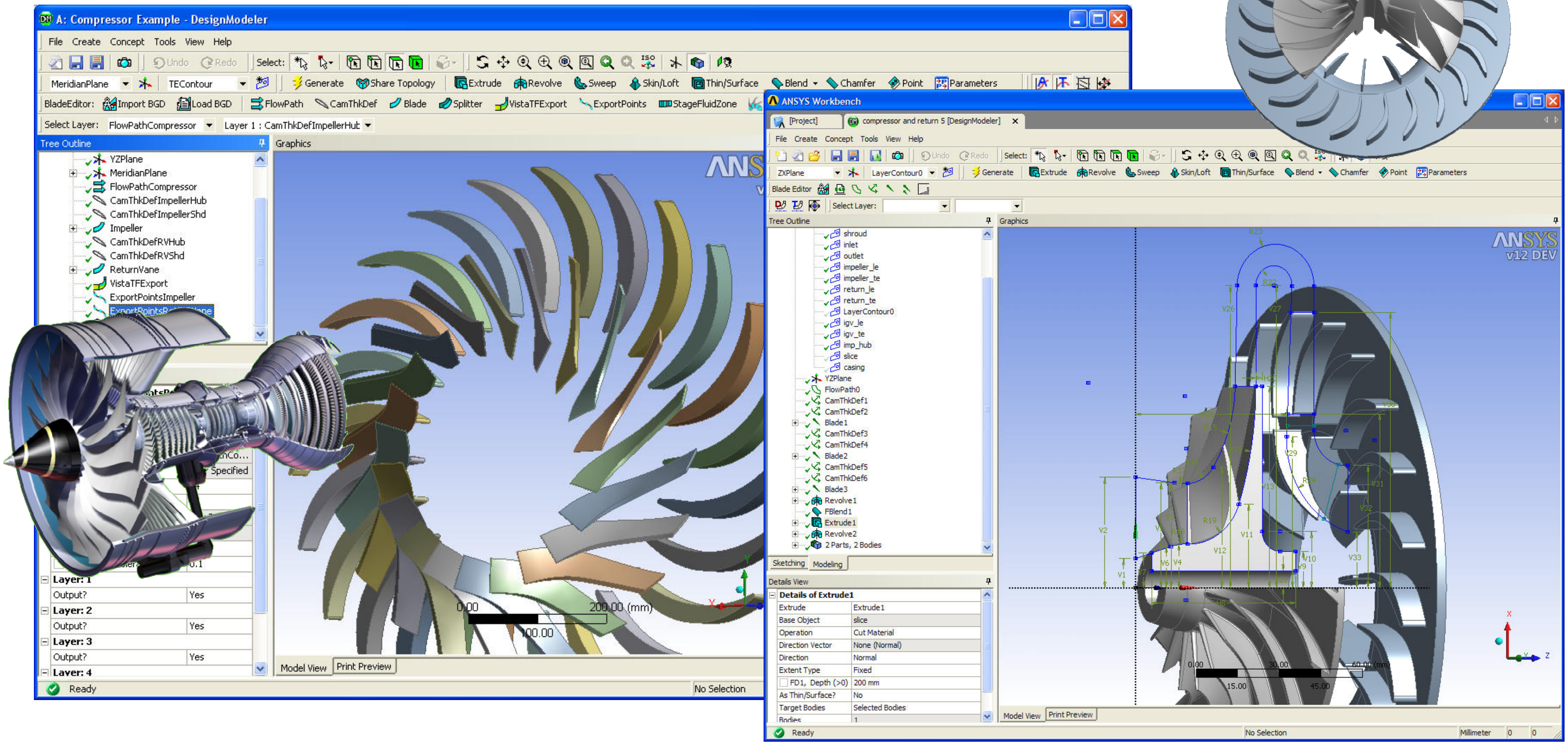
RDO Centrifugal Compressor

Parameterization

Parametric geometry definition using ANSYS BladeModeler (17 geometric parameter)

Model completion and meshing using ANSYS Workbench



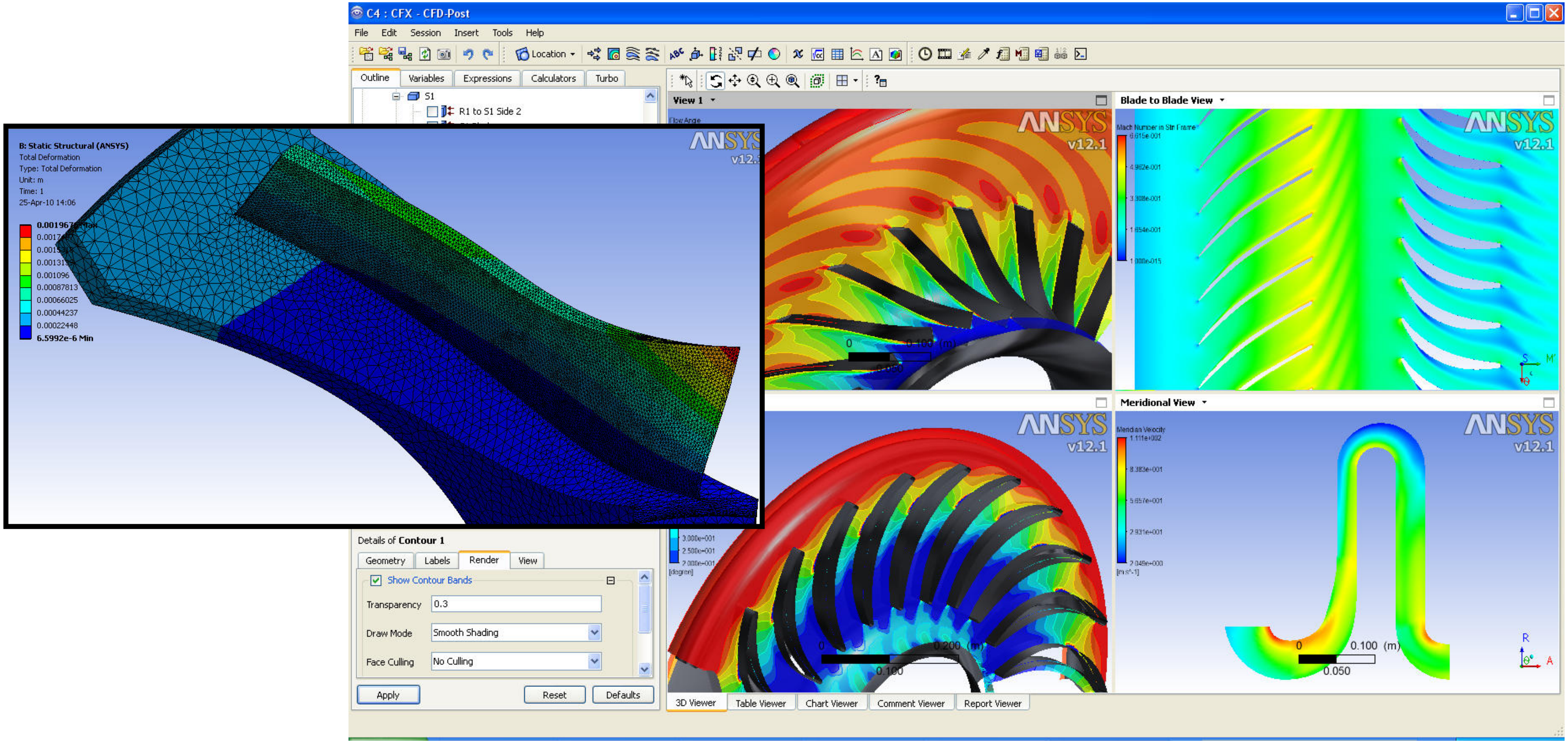


RDO Centrifugal Compressor

Fluid Structure Interaction (FSI) coupling

Parametric fluid simulation setup using ANSYS CFX

Parametric mechanical setup using ANSYS Workbench



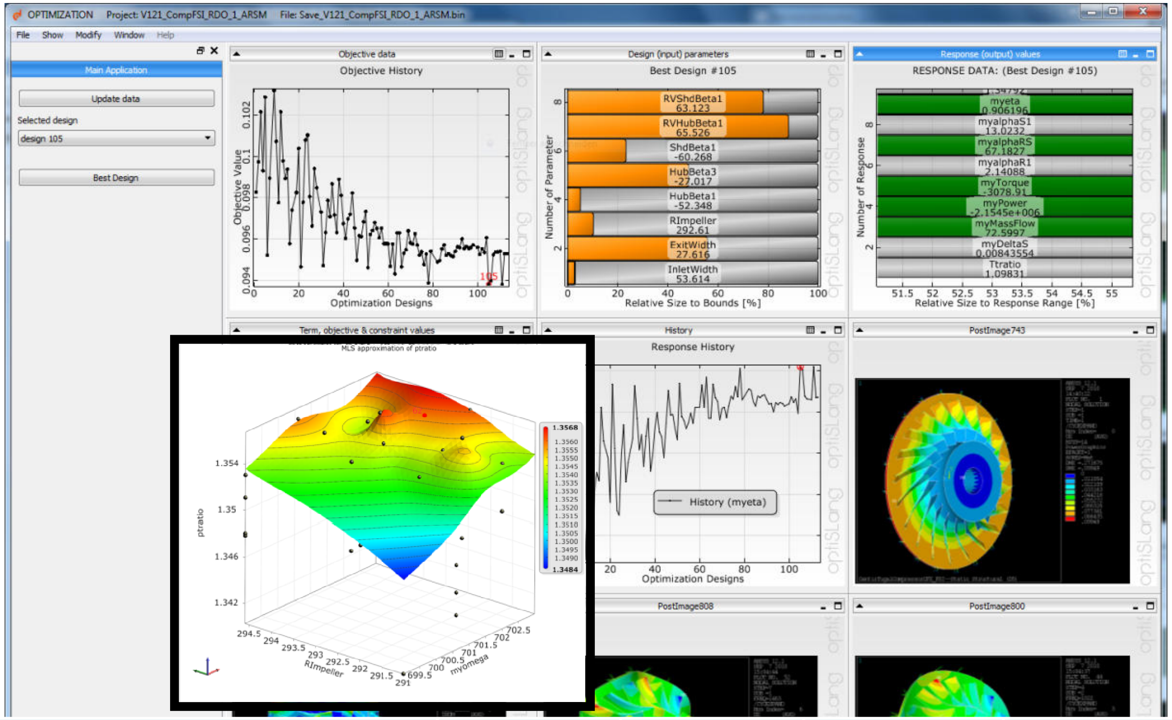
by courtesy of ANSYS

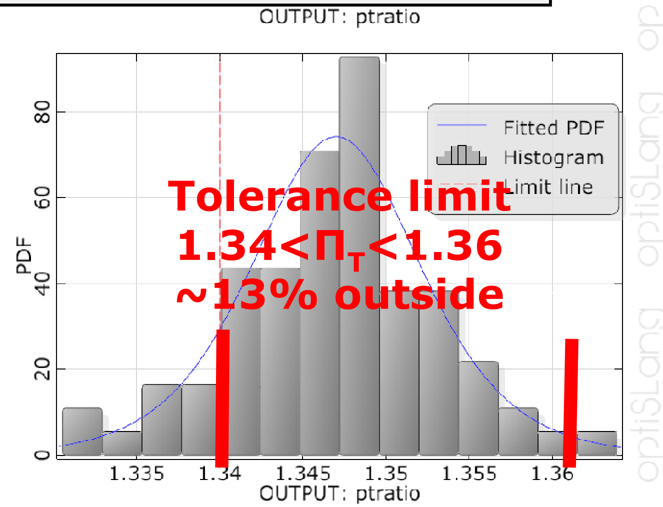
RDO Centrifugal Compressor

Optimization goal: increase efficiency

Constraints: 2 pressure ratio’s, 66 frequency constraints, Robustness

Input Parameter	21
Output Parameter	43
Constraints	68





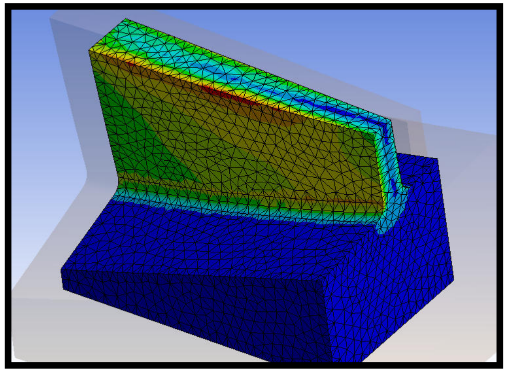
Statistic data		
	Max:	1.364
	Sigma:	0.006107
	Kurtosis:	3.589
Fitted PDF: Logistic		
	Sigma:	0.006107
Limit x = 1.34		
	P_fit =	0.110889

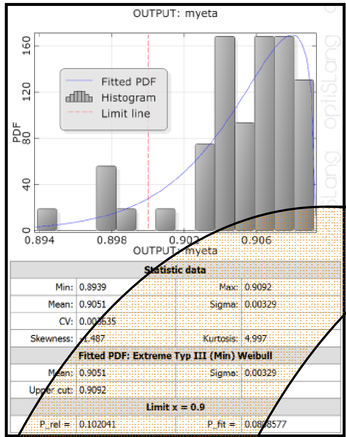
	Initial	SA	ARSM I	EA I	ARSM II	ARSM III
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356	1.351
Efficiency [%]	86.72	89.15	90.62	90.67	90.76	90.73
#Designs	-	100	105	84	62	40

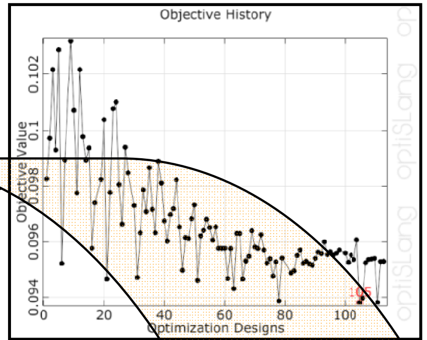
by courtesy of ANSYS

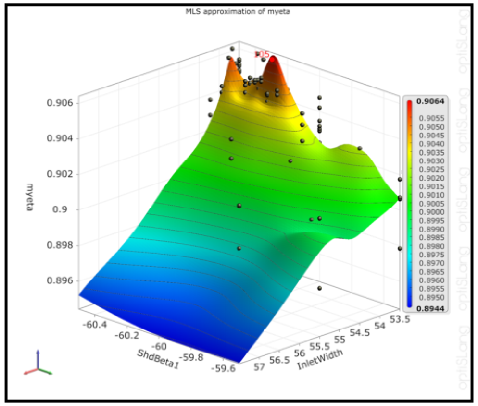
RDO Centrifugal Compressor

Robust Design Optimization with respect to 21 design parameters and 20 random geometry parameters, including manufacturing tolerances.
Robust Design was reached after 400+250=650 design evaluations.



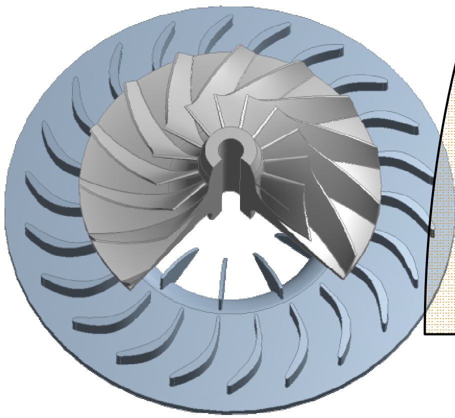


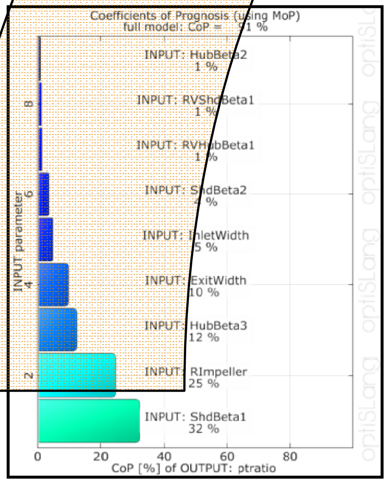


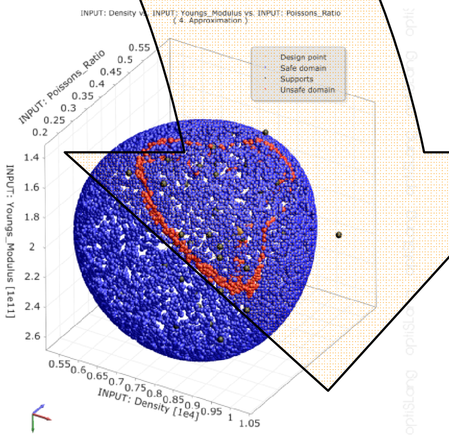


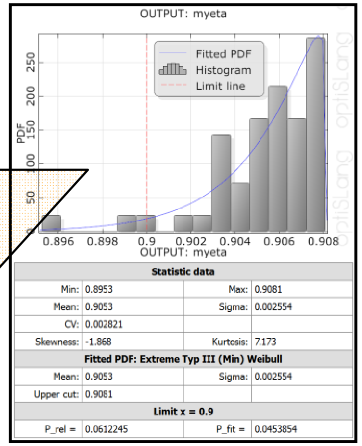
Robustness evaluation

RDO optimization









Sensi + first optimization step

Robustness proof using Reliability Analysis

by courtesy of ANSYS

dynardo

Benefits of Robust Design Optimization

Identify product design parameters which are critical for the achievement of a performance characteristic!

- Quantify the effect of variations on product behavior and performance
- Adjust the design parameter to hit the target performance
  - ✓Reduces product costs

- Understanding potential sources of variations
- Minimize the effect of variations (noise)
  - ✓More robust and affordable designs

- Cost-effective quality inspection
  - ✓No inspection for parameters that are not critical for the performance